SMEFT beyond $\mathcal{O}(1/\Lambda^2)$

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based on 2001.01453 with A. Helset and M. Trott (NBI)
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Oklahoma State, Mar 18th 2021

Motivation

No obvious signs of new light states at LHC — parametrize BSM effects with SM-EFT = SMEFT

$$\mathscr{L} = \mathscr{L}_{SM} + \sum_d \sum_i \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathscr{O}_i^{(d)}(Q, u_c, d_c, L, e_c, H, D_\mu, F_{\mu\nu} \cdots)$$

write down all operators, lowest mass dimension terms dominate in the IR

Odd dimensions always violate B or L, so focus has been on dim-6 (~60 operators)

Motivation

$$
\begin{aligned}
\mathcal{Q}_{Hl}^{(1)} &= (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{l}\gamma^\mu l) \\
\mathcal{Q}_{He} &= (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e) \\
\mathcal{Q}_{Hq}^{(1)} &= (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q) \\
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\mathcal{Q}_{Hd} &= (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)
\end{aligned}
$$

Integration by parts (IBP) or field redefinitions (EOM redundancy)

$$\phi_i \to \phi_i + \frac{\delta\phi_i}{\Lambda^2} + \cdots$$

reshuffle operators but don't change physics

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$$\phi_i \to \phi_i + \frac{\delta\phi_i}{\Lambda^2} + \cdots$$

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Ex: $B_\mu \to B_\mu + \frac{2}{g'}\frac{C_{H\ell}^{(1)}}{\Lambda^2}(\bar{L}\gamma_\mu L)$ removes $\mathcal{Q}_{H\ell}^{(1)}$ in favor of $\mathcal{Q}_{Hq}^{(1)}$, $\mathcal{Q}_{He}$, $\mathcal{Q}_{Hu}$, $(\partial^\rho B_{\rho\mu})(\bar{L}\gamma^\mu L)$, etc.

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Therefore: SMEFT analysis requires working with a complete 'basis' of operators. Often "Warsaw basis"

Motivation

Z,W couplings

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\end{aligned}
$$

Bhabha scattering

$$
\begin{aligned}
\mathcal{Q}_{ee} &= (\bar{e}\gamma^\mu e)(\bar{e}\gamma^\mu e)\\
\mathcal{Q}_{le} &= (\bar{l}\gamma^\mu l)(\bar{e}\gamma^\mu e)\\
\mathcal{Q}_{ll} &= (\bar{l}_p\gamma^\mu l_p)(\bar{l}_r\gamma^\mu l_r)
\end{aligned}
$$

$$
\mathcal{Q}_W = \varepsilon_{ijk} W_\mu^{i\nu} W_\nu^{j\rho} W_\rho^{k\mu}
$$

TGC/multi-boson

Top data

$$
\begin{aligned}
\mathcal{Q}^{(1)}_{\substack{qq\\prst}} &= (\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma_\mu q_t),\\
\mathcal{Q}^{(3)}_{\substack{qq\\prst}} &= (\bar{q}_p\gamma^\mu \tau^I q_r)(\bar{q}_s\gamma_\mu \tau_I q_t),\\
\mathcal{Q}_{\substack{uu\\prst}} &= (\bar{u}_p\gamma^\mu u_r)(\bar{u}_s\gamma_\mu u_t),\\
\mathcal{Q}^{(1)}_{\substack{ud\\prst}} &= (\bar{u}_p\gamma^\mu u_r)(\bar{d}_s\gamma_\mu d_t),\\
\mathcal{Q}^{(8)}_{\substack{ud\\prst}} &= (\bar{u}_p\gamma^\mu T^A u_r)(\bar{d}_s\gamma_\mu T^A d_t)
\end{aligned}
$$

Operators impact multiple processes: Global approach needed

$$
\begin{aligned}
\mathcal{Q}_{HD} &= (D_\mu H^\dagger H)(H^\dagger D^\mu H)\\
\mathcal{Q}_{HWB} &= (H^\dagger \sigma^i H) W^i_{\mu\nu} B^{\mu\nu}\\
\mathcal{Q}_{Hl}^{(3)} &= (iH^\dagger \overleftrightarrow{D}^i_\mu H)(\bar{l}\sigma^i\gamma^\mu l)\\
\mathcal{Q}'_{ll} &= (\bar{l}_p\gamma^\mu l_r)(\bar{l}_r\gamma^\mu l_p)
\end{aligned}
$$

input quantities

$$
\begin{aligned}
\mathcal{Q}_{Hbox} &= (H^\dagger H)\,\square\,(H^\dagger H)\\
\mathcal{Q}_{HG} &= (H^\dagger H) G^a_{\mu\nu} G^{a\mu\nu}\\
\mathcal{Q}_{HB} &= (H^\dagger H) B_{\mu\nu} B^{\mu\nu}\\
\mathcal{Q}_{HW} &= (H^\dagger H) W^i_{\mu\nu} W^{i\mu\nu}\\
\mathcal{Q}_{uH} &= (H^\dagger H)(\bar{q}\tilde{H}u)\\
\mathcal{Q}_{dH} &= (H^\dagger H)(\bar{q}Hd)\\
\mathcal{Q}_{eH} &= (H^\dagger H)(\bar{q}e)\\
\mathcal{Q}_G &= \varepsilon_{abc} G_\mu^{a\nu} G_\nu^{b\rho} G_\rho^{c\mu}\\
\mathcal{Q}_{uG} &= (\bar{q}\sigma^{\mu\nu} T^a \tilde{H} u) G^a_{\mu\nu}
\end{aligned}
$$

H processes

Pattern in deviations informs about new physics scale and type

Motivation

How does SMEFT contribute? State of the art:

$$|A|^2 = |A_{SM}|^2 + \frac{2Re(A_{SM}\,A_6)}{\Lambda^2} + \frac{|A_6|^2}{\Lambda^4} + \cdots$$

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"New physics squared"; Higher order in $1/\Lambda$

Why would you ever go beyond $1/\Lambda^2$?

- Uncertainty: To know error on $1/\Lambda^2$ piece, we should know next order

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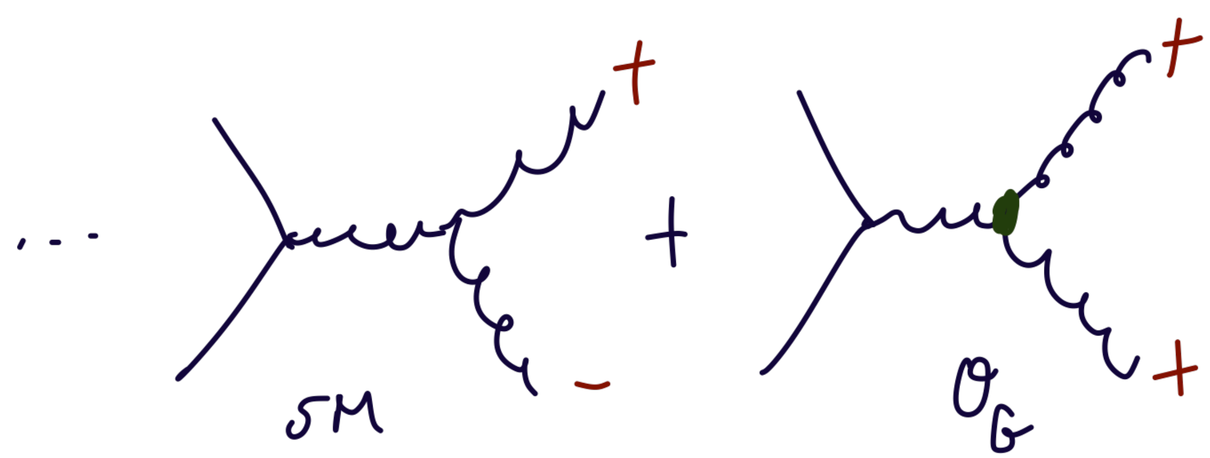
- Interference can be suppressed: e.g. if there is a mismatch in the helicity of the SM and the $1/\Lambda^2$ operators

Classic example: $\mathcal{O}_G = f_{ABC} G^A_{\mu\nu} G^{B,\nu\rho} G^{C,\mu}_{\rho}$ and dijets



SM and $\mathcal{O}_G$ produce different helicity gluons!

No interference, so first effect is at $(\mathcal{O}_G)^2$

[Shadmi, Dixon '93]

- Energy considerations:

by dimensional analysis:

$$\frac{2\,Re(A_{SM}^{*}A_{6})}{\Lambda^{2}} \sim \frac{E^{2}}{\Lambda^{2}} \quad \left(\text{or } \frac{v^{2}}{\Lambda^{2}}\right)$$

$$\frac{\left|A_{6}\right|^{2}}{\Lambda^{4}} \sim \frac{E^{4}}{\Lambda^{4}} \quad \left(\text{or } \frac{v^{4}}{\Lambda^{4}}, \frac{v^{2}E^{2}}{\Lambda^{4}}\right)$$

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For high energy measurement (LHC, tails of kinematic distributions), $1/\Lambda^4$ increasingly important

OK, lets just include new physics squared piece

$$|A|^2 = |A_{SM}|^2 + \frac{2Re(A_{SM}\,A_6)}{\Lambda^2} + \frac{|A_6|^2}{\Lambda^4} + \cdots$$

easy, known, but not the whole story

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But (dim-6)² is the same order in $1/\Lambda$ as dim-8 effects interfering with SM…

$$\frac{|A_6|^2}{\Lambda^4} + 2\frac{Re(A^*_{SM}A_8)}{\Lambda^4} + \cdots$$

so both should be included to expand consistently

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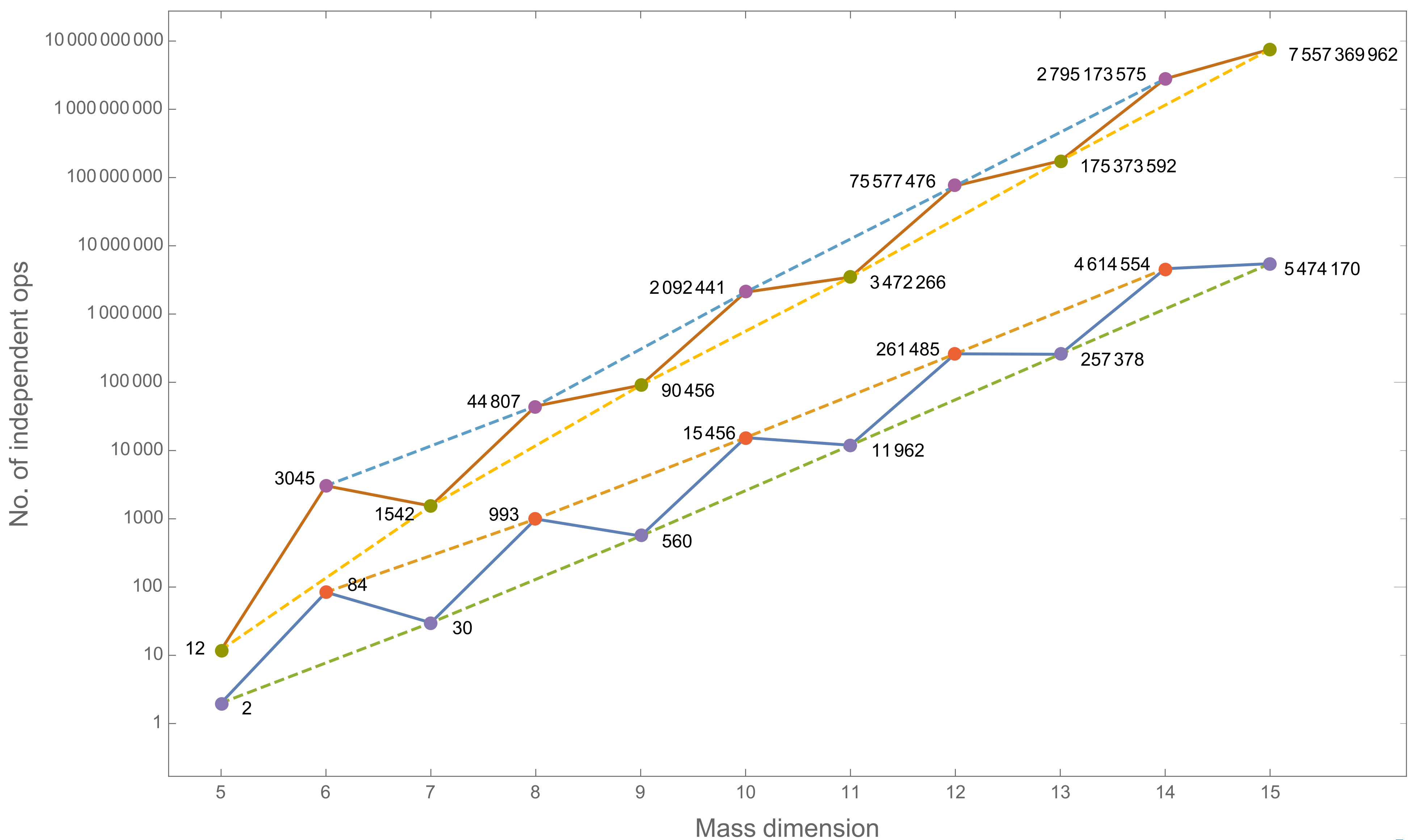
Extending to dim-8 introduces 993 more operators, even assuming fermion flavor universality!

Theoretically: need to know how 993 new effects enter?!
Experimentally: means 993 more measurements needed ?!

Going even further…

operators at given mass dimension known (Hilbert series method), but explodes with mass dim!



[Henning et al 1512.03433]

So:

Q: Is it phenomenologically viable to go beyond dimension-6?

Q: Does it make a difference? Meaning, e.g. does (dim-6)^2 suffice to capture $1/\Lambda^4$ effects?

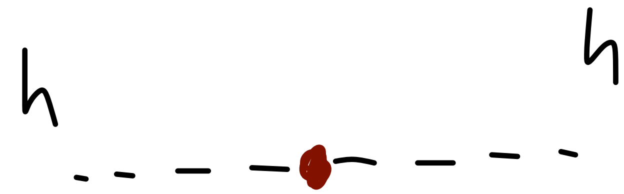
What do these operators actually do?

$\mathcal{O}_{8,HD}$	$(H^\dagger H)^2(D_\mu H^\dagger\, D^\mu H)$	$\mathcal{O}_{8,DH\tilde{W}3b}$	$\epsilon_{IJK}\,(D^\mu H^\dagger \tau^I D^\nu H)(W^J_{\mu\rho}\widetilde{W}^{\rho,K}_\nu + \widetilde{W}^J_{\mu\rho}W^{\rho,K}_\nu)$
$\mathcal{O}_{8,HD2}$	$\delta_{IJ}\,(H^\dagger H)(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^J D_\mu H)$	$\mathcal{O}_{8,DHWB}$	$\delta_{IJ}\,(D^\mu H^\dagger\, \tau^I D_\mu H)B^{\rho\sigma}W^J_{\rho\sigma}$
$\mathcal{O}_{8,DHB}$	$(D^\mu H^\dagger\, D^\nu H)B_{\mu\rho}B^\rho_\nu$	$\mathcal{O}_{8,DH\tilde{W}B}$	$\delta_{IJ}\,(D^\mu H^\dagger\, \tau^I D_\mu H)B^{\rho\sigma}\widetilde{W}^J_{\rho\sigma}$
$\mathcal{O}_{8,DHB2}$	$(D^\mu H^\dagger D_\mu H)B^{\rho\sigma}B_{\rho\sigma}$	$\mathcal{O}_{8,DHWB2}$	$i\,\delta_{IJ}\,(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho}W^{\rho,J}_\nu - B_{\nu\rho}W^{\rho,J}_\mu)$
$\mathcal{O}_{8,DH\tilde{B}2}$	$(D^\mu H^\dagger D_\mu H)B^{\rho\sigma}\widetilde{B}_{\rho\sigma}$	$\mathcal{O}_{8,DHWB3}$	$\delta_{IJ}\,(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho}W^{\rho,J}_\nu + B_{\nu\rho}W^{\rho,J}_\mu)$
$\mathcal{O}_{8,DHG}$	$\delta_{AB}\,(D^\mu H^\dagger D^\nu H)G^A_{\mu\rho}G^{\rho,B}_\nu$	$\mathcal{O}_{8,DH\tilde{W}B2}$	$\delta_{IJ}\,(D^\mu H^\dagger \tau^I D^\nu H)(B^\rho_{[\mu}\widetilde{W}^J_{\nu]\rho} - \widetilde{B}^\rho_{[\mu}W^J_{\nu]\rho})$
$\mathcal{O}_{8,DHG2}$	$\delta_{AB}\,(D^\mu H^\dagger D_\mu H)G^{\rho\sigma,A}G^B_{\rho\sigma}$	$\mathcal{O}_{8,DH\tilde{W}B3}$	$\delta_{IJ}\,(D^\mu H^\dagger \tau^I D^\nu H)(B^\rho_{\{\mu}\widetilde{W}^J_{\nu\}\rho} + \widetilde{B}^\rho_{\{\mu}W^J_{\nu\}\rho})$
$\mathcal{O}_{8,DH\tilde{G}2}$	$\delta_{AB}\,(D^\mu H^\dagger D_\mu H)G^{\rho\sigma,A}\widetilde{G}^B_{\rho\sigma}$	$\mathcal{O}_{8,HDHB}$	$i\,(H^\dagger H)(D_\mu H^\dagger\, D_\nu H)B^{\mu\nu}$
$\mathcal{O}_{8,DHW}$	$\delta_{IJ}\,(D^\mu H^\dagger D^\nu H)W^I_{\mu\rho}W^{\rho,J}_\nu$	$\mathcal{O}_{8,HDH\tilde{B}}$	$i\,(H^\dagger H)(D_\mu H^\dagger\, D_\nu H)\widetilde{B}^{\mu\nu}$
$\mathcal{O}_{8,DHW2}$	$\delta_{IJ}\,(D^\mu H^\dagger D_\mu H)W^{\rho\sigma,I}W^J_{\rho\sigma}$	$\mathcal{O}_{8,HDHW}$	$i\,\delta_{IJ}\,(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H)W^J_{\mu\nu}$
$\mathcal{O}_{8,DH\tilde{W}2}$	$\delta_{IJ}\,(D^\mu H^\dagger D_\mu H)W^{\rho\sigma,I}\widetilde{W}^J_{\rho\sigma}$	$\mathcal{O}_{8,HDH\tilde{W}}$	$i\,\delta_{IJ}\,(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H)\widetilde{W}^J_{\mu\nu}$
$\mathcal{O}_{8,DHW3}$	$\epsilon_{IJK}\,(D^\mu H^\dagger \tau^I D^\nu H)W^J_{\mu\rho}W^{\rho,K}_\nu$	$\mathcal{O}_{8,HDHW2}$	$i\,\epsilon_{IJK}\,(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^J D^\nu H)W^K_{\mu\nu}$
$\mathcal{O}_{8,DH\tilde{W}3a}$	$\epsilon_{IJK}\,(D^\mu H^\dagger \tau^I D^\nu H)(W^J_{\mu\rho}\widetilde{W}^{\rho,K}_\nu - \widetilde{W}^J_{\mu\rho}W^{\rho,K}_\nu)$	$\mathcal{O}_{8,HDH\tilde{W}2}$	$i\,\epsilon_{IJK}\,(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^J D^\nu H)\widetilde{W}^K_{\mu\nu}$

a subset of the bosonic operators at dim-8….

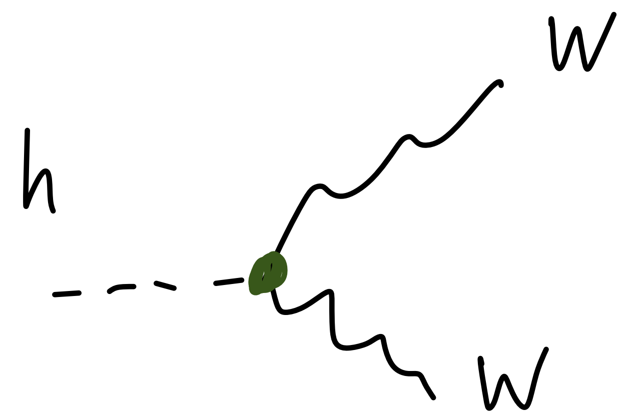
What do these operators actually do?

Change field strength normalization/inputs



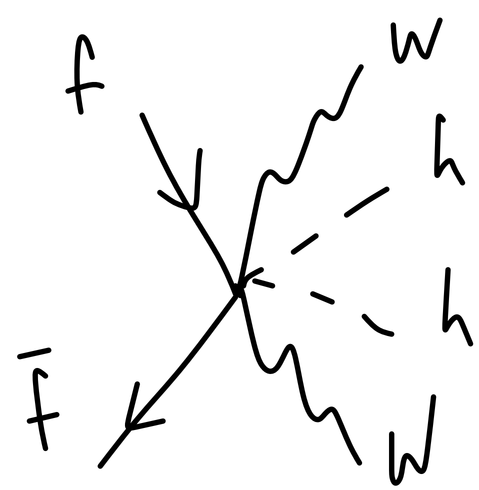
ex.) $(H^\dagger H)\,\Box\,(H^\dagger H)$

Modify existing vertices



ex.) $(H^\dagger H)\, W^a_{\mu\nu} W^{a,\mu\nu}$

New multi-particle interactions



ex.) $(\bar{\psi}\psi)^2$

Punchline of this talk

Its possible to reorganize SMEFT operators (= find a basis), where

2 and 3-particle interactions are sensitive to the minimal number of operators

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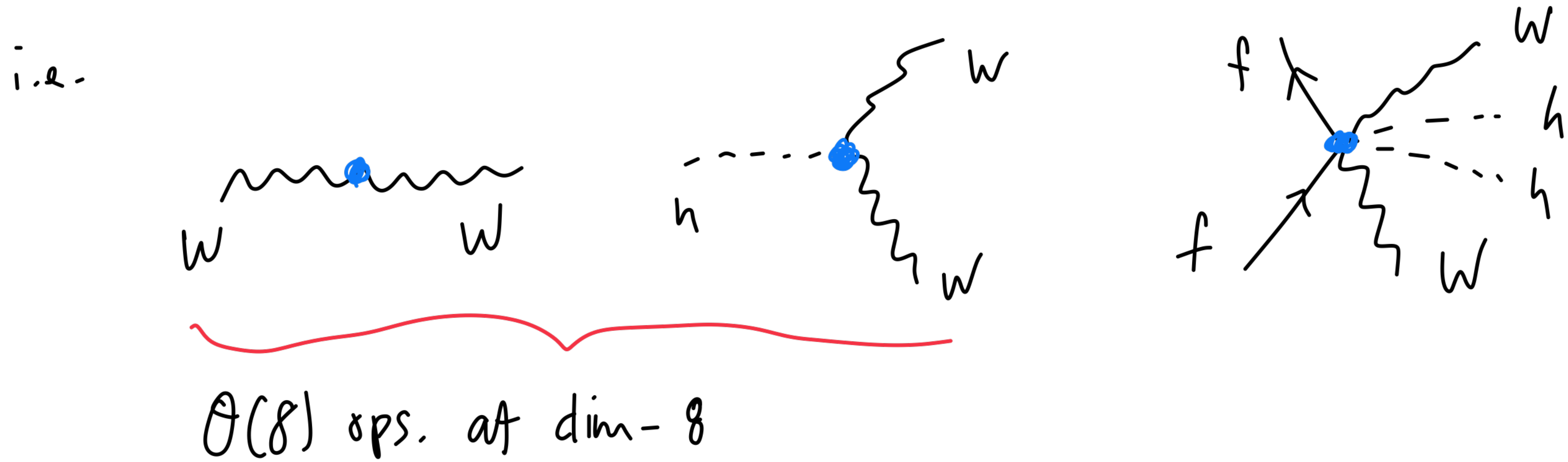

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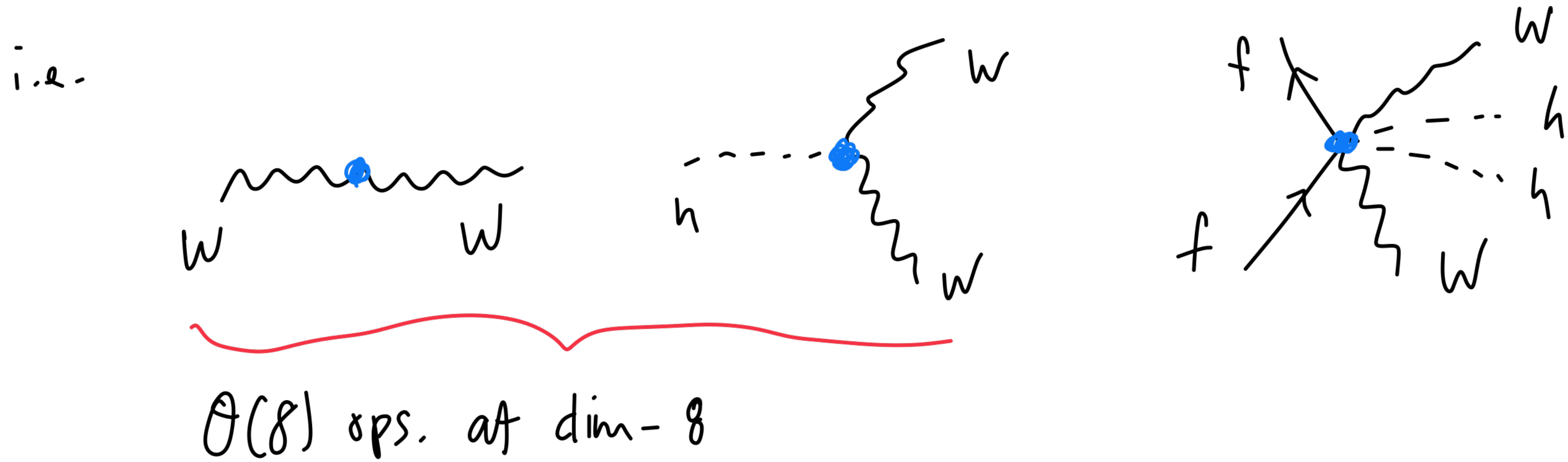

With fewer operators around, can actually do complete $1/\Lambda^4$ calculations for certain processes.

Use those processes as simple laboratories for 'truncation error studies'

Or, to answer questions posed:

Q: Is it phenomenologically viable to go beyond dimension-6?

For resonant (h/W/Z/t) phenomenology involving 2- and 3-point vertices, yes

Q: Does it make a difference?

Absolutely, especially when applied to loop-level SM processes

First hint: Misiak et al 1812.11513

Fully exploiting IBP and EOM redundancies, the only SMEFT operator types that contribute to bosonic 2-pt interactions are:

$$H^n\,,\ H^n X^2\,,\ D^2 H^n$$

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Why not e.g. D^4H^4 ? $\left(DH \sim \partial h + igAv + igAh\right)$

- $(DH^\dagger)(DH)(DH^\dagger)(DH)$? — too many fields
- $(D_{\{\mu\nu\}}H^\dagger D_{\{\mu\nu\}}H)(H^\dagger H)$? — via IBP and EOM, reduces to operators with 2 derivs + operators with > 2 fields

...

Similar arguments can be made for operators with field strengths, more derivatives

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$$H^n\,,\; H^n X^2\,,\; D^2 H^n$$

Bosonic kinetic piece defined by two functions:

$$h(H)(D_\mu H^\dagger D_\mu H)\,,\; g_{AB}(H)\mathscr{W}^A_{\mu\nu}\mathscr{W}^{B\mu\nu}$$

$$\mathscr{W}^A = \left(W^1, W^2, W^3, B\right)$$

this choice defines a basis

Even better:

Number of H^n, $H^n X^2$, $D^2 H^n$ type operators ~ doesn't change with mass dimension

	Mass Dimension				
Field space connection	6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4

Consequence of group theory + Bose statistics

contributions to h_{IJ}

$$Q_{HD}^{(8+2n)} = \left(H^\dagger H\right)^{n+2}\left(D_\mu H\right)^\dagger (D^\mu H)$$

$$Q_{H,D2}^{(8+2n)} = \left(H^\dagger H\right)^{n+1}\left(H^\dagger \sigma_a H\right)\left(D_\mu H\right)^\dagger \sigma^a (D^\mu H)$$

Example operator counting:

$(H^\dagger H)^n\, W_L^2$ ignore Lorentz, focus on $SU(2)_W$ reps.

$H = (1/2) \;\therefore\; H^n = (n/2)$

$H^\dagger = (1/2) \;\therefore\; (H^\dagger)^n = (n/2)$

$W_L^2 = (0 \oplus 2)$ enforced by Bose symm.

$(H^\dagger H)^n = (0 \oplus 1 \oplus 2 \oplus \ldots n) \quad \otimes \quad W_L^2 = (0 \oplus 2)$ = 2 invariants

[+1 for B_L^2 and +1 for $W_L B_L$ = 4]

To get $SU(2)_W$ 2, need $\geq$ 4 Higgses $\rightarrow$ operator dimension ≥ 8

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[+1 for B_L^2 and +1 for $W_L B_L$ = 4]

To get SU(2)$_W$ 2, need ≥ 4 Higgses → operator dimension ≥ 8

contributions to g$_{AB}$

$$Q_{HB}^{(6+2n)} = (H^\dagger H)^{n+1} B^{\mu\nu} B_{\mu\nu},$$
$$Q_{HW}^{(6+2n)} = (H^\dagger H)^{n+1} W_a^{\mu\nu} W_{\mu\nu}^a,$$
$$Q_{HWB}^{(6+2n)} = (H^\dagger H)^{n} (H^\dagger \sigma^a H) W_a^{\mu\nu} B_{\mu\nu},$$
$$Q_{HW,2}^{(8+2n)} = (H^\dagger H)^{n} (H^\dagger \sigma^a H)(H^\dagger \sigma^b H) W_a^{\mu\nu} W_{b,\mu\nu},$$

Convenient to work with real fields: $H(\phi_I) = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$

Can rewrite scalar quadratic form as a metric in field space $h_{IJ}(\phi)\left(D_\mu\phi\right)^I\left(D_\mu\phi\right)^J$

$$h_{IJ} = \left[1 + \phi^2 C^{(6)}_{H\Box} + \sum_{n=0}^{\infty}\left(\frac{\phi^2}{2}\right)^{n+2}\left(C^{(8+2n)}_{HD} - C^{(8+2n)}_{H,D2}\right)\right]\delta_{IJ} + \frac{\Gamma^I_{A,J}\phi_K\Gamma^K_{A,L}\phi^L}{2}\left(\frac{C^{(6)}_{HD}}{2} + \sum_{n=0}^{\infty}\left(\frac{\phi^2}{2}\right)^{n+1} C^{(8+2n)}_{H,D2}\right)$$

SU(2) generators for real fields

SM, $g_{AB}, h_{IJ} = \mathbf{1}$. Including higher dimension operators, field space metrics become curved $\longrightarrow$ 'geometric' SMEFT or `geoSMEFT'

[Burgess, Lee, Trott '10, Alonso, Jenkins, Manohar '15, '16, Helset, Paraskevas, Trott 1803.08001]

What about 3-pt interactions? Similar story

- 3 fields only, Lorentz invariance
- non-Higgs derivatives increase field count or introduce momentum

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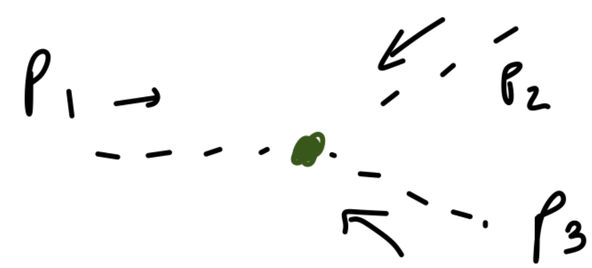
But **all** momentum dot products reduce to masses once we impose momentum conservation

Ex.) $D_\mu H \, (D^\mu \bar{\psi}) \, \psi$

$$\sim (p_H \cdot p_{\bar{\psi}}) \, H \, \bar{\psi} \, \psi$$

$$\sim \left(\frac{m_\psi^2 - m_H^2 - m_{\bar{\psi}}^2}{2} \right) H \, \bar{\psi} \, \psi$$



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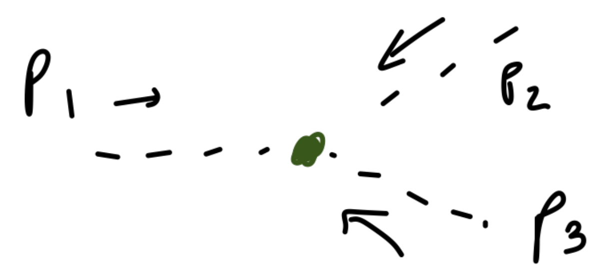

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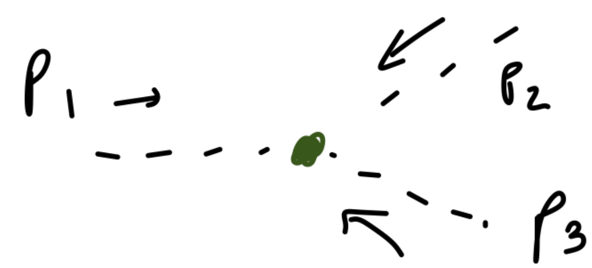

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What about 3-pt interactions? Similar story

Net result: limited options

- $DF_1\, DF_2\, DF_3\, DF_4$ ✗
- $(DX)^2\, H^2$ ✗
- $H^2\, X^3$ ✓
- $(D\bar{\psi})\,\psi\,(DH)\,H$ ✗
- $\bar{\psi}\psi\,(DH)\,H^3$ ✓

...

exactly the ‘special 3-body kinematics’ story from on-shell amplitude-land

Allowed 3-pt structures:

$$h_{IJ}(\phi)(D_\mu\phi)^I(D_\mu\phi)^J, \quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$$

$$k^A_{IJ}(\phi)(D_\mu\phi)^I(D_\nu\phi)^J\,\mathcal{W}^{\mu\nu}_A, \quad f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho,$$

[+ versions with G^A]

$$Y(\phi)\bar{\psi}_1\psi_2, \quad L_{I,A}(\phi)\bar{\psi}_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I, \quad d_A(\phi)\bar{\psi}_1\sigma^{\mu\nu}\psi_2\mathcal{W}^A_{\mu\nu},$$

Higgs-dependent 'connections'

As before, # operators small and remains ~fixed for increasing mass dimension

	Mass Dimension				
Field space connection	6	8	10	12	14
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$

Example: $L_{I,A}(\phi)\bar{\psi}_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I$

contributing operators

$$\mathcal{Q}^{1,(6+2n)}_{\substack{H\psi\\pr}} = (H^\dagger H)^n H^\dagger \, i\overleftrightarrow{D}^\mu H \bar{\psi}_p\gamma_\mu\psi_r,$$

$$\mathcal{Q}^{3,(6+2n)}_{\substack{H\psi\\pr}} = (H^\dagger H)^n \, H^\dagger \, i\overleftrightarrow{D}^\mu_a H \bar{\psi}_p\gamma_\mu\sigma_a\psi_r,$$

higher dim. versions of "class 7" operators

$$\mathcal{Q}^{2,(8+2n)}_{\substack{H\psi\\pr}} = (H^\dagger H)^n (H^\dagger\sigma_a H) \, H^\dagger \, i\overleftrightarrow{D}^\mu H \bar{\psi}_p\gamma_\mu\sigma_a\psi_r,$$

$$\mathcal{Q}^{\epsilon,(8+2n)}_{\substack{H\psi\\pr}} = \epsilon^a_{bc}\,(H^\dagger H)^n\,(H^\dagger\sigma_c H)\,H^\dagger\, i\overleftrightarrow{D}^\mu_b H \bar{\psi}_p\gamma_\mu\sigma_a\psi_r.$$

new effects from $d \geq 8$

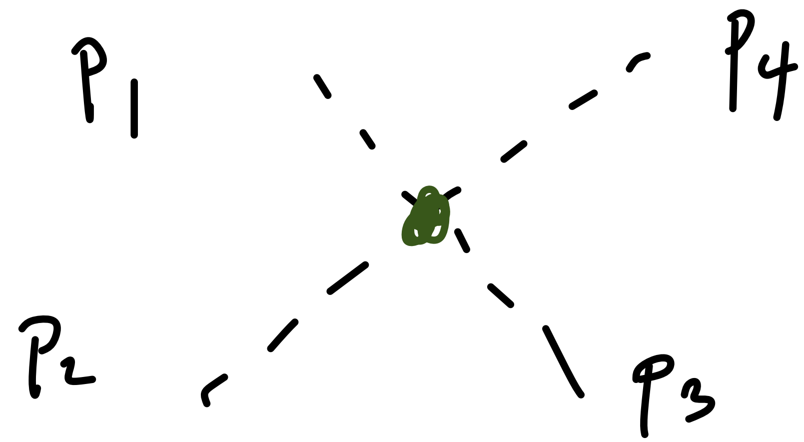
compact form for connection:

$$L^{\psi,pr}_{J,A} = -(\phi\,\gamma_4)_J\delta_{A4}\sum_{n=0}^\infty C^{1,(6+2n)}_{\substack{H\psi\\pr}}\left(\frac{\phi^2}{2}\right)^n - (\phi\,\gamma_A)_J(1-\delta_{A4})\sum_{n=0}^\infty C^{3,(6+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n$$

$$+\frac{1}{2}(\phi\,\gamma_4)_J(1-\delta_{A4})\left(\phi_K\Gamma^K_{A,L}\phi^L\right)\sum_{n=0}^\infty C^{2,(8+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n$$

$$+\frac{\epsilon^A_{BC}}{2}(\phi\,\gamma_B)_J\left(\phi_K\Gamma^K_{C,L}\phi^L\right)\sum_{n=0}^\infty C^{\epsilon,(8+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n$$

4-pt interactions: can we go 'full metric'?



Key part of 2- and 3-pt result is that special kinematics forbade

$$D \sim \textbf{momentum}$$

No longer true at ≥ 4-pt interactions. Operators can depend on

$$\mathcal{O} \sim s^n\, t^m$$

$\longrightarrow$ infinite set of higher derivative operators can contribute

geoSMEFT at work:

SMEFT phenomenology for processes involving 2, 3-pt interactions now doable to any order in v^2/Λ^2

Specifically, $\mathcal{O}(1/\Lambda^4)$ easily calculated for a large set of processes

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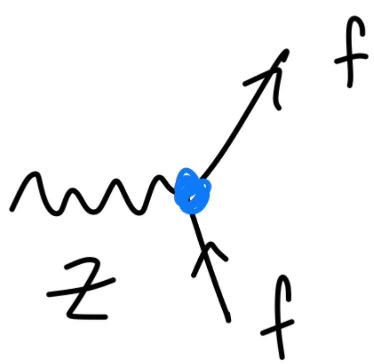

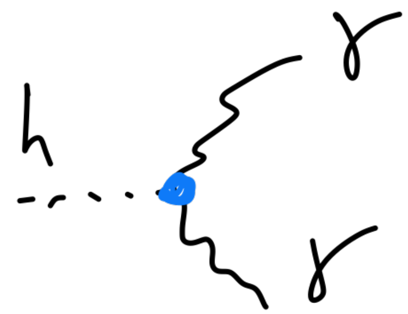

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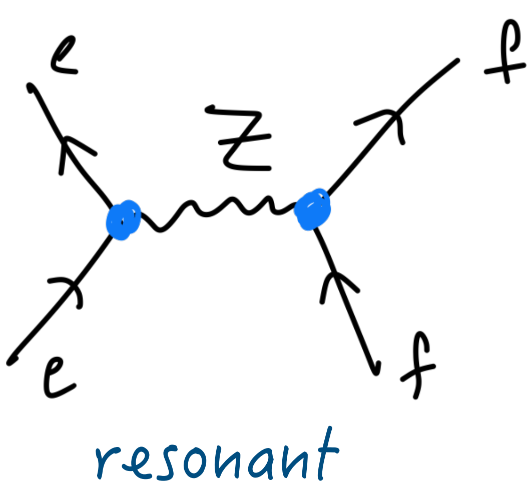

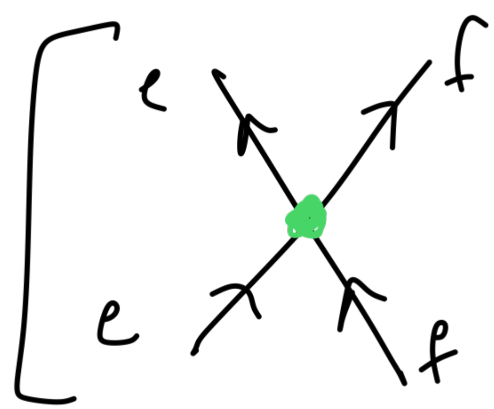

suppressed by $\frac{\Gamma_Z m_Z}{v^2}$

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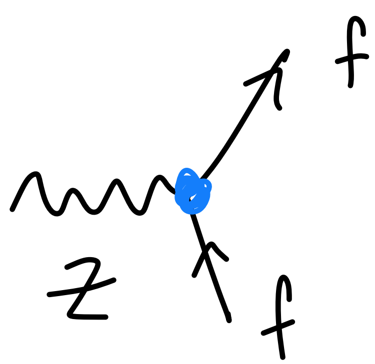
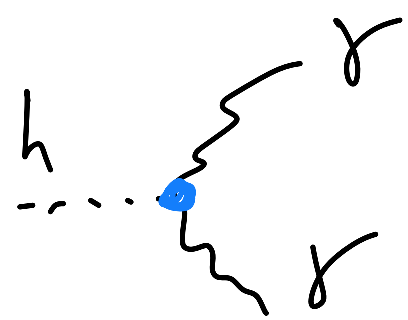
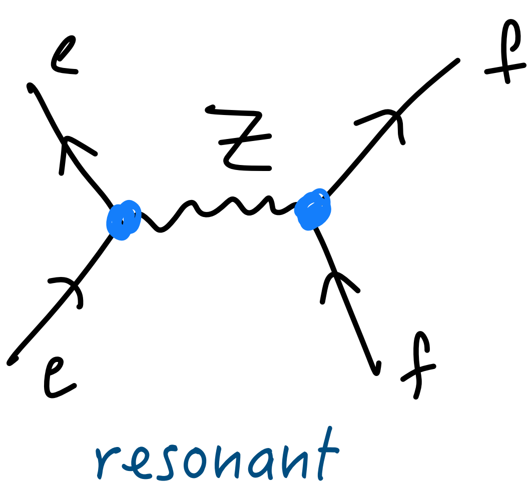
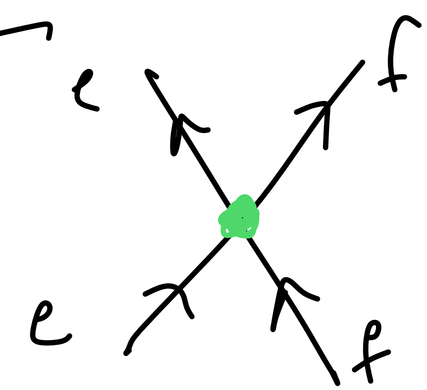




[2007.00565 Hays, Helset, AM, Trott]

and



[

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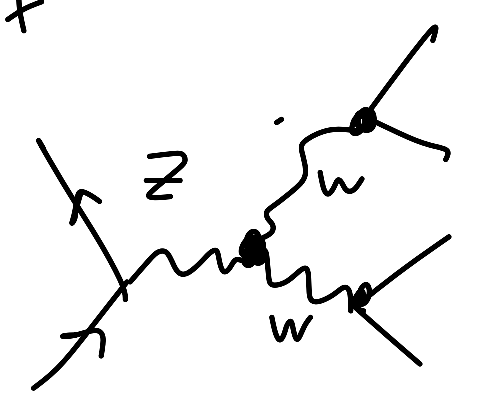
(also 

)

[2102.02819 Corbett, Helset, AM, Trott]

What can we do with this? `EW inputs’

Bosonic kinetic terms used to define the gauge boson mass basis

$$W^3_\mu, B_\mu \longrightarrow A_\mu, Z_\mu$$

& couplings to mass eigenstates define: $e,\ g_Z,\ \sin^2\theta_Z$

$$D_\mu \psi = \left[\partial_\mu + i\bar{g}_3\,\mathcal{G}^\mu_{\mathcal{A}}\,T^{\mathcal{A}} + i\frac{\bar{g}_2}{\sqrt{2}}\left(\mathcal{W}^+\,T^+ + \mathcal{W}^-\,T^-\right) + i\bar{g}_Z\left(T_3 - s^2_{\theta_Z} Q_\psi\right)\mathcal{Z}^\mu + i\,Q_\psi\,\bar{e}\,\mathcal{A}^\mu\right]\psi.$$

SM: $e,\ g_Z,\ \sin^2\theta_Z$ =
functions of g, g' alone

SMEFT: relation altered by operators
that feed into kinetic terms:

ex.) $C^{(6)}_{HW}\, H^\dagger H\, W^A_{\mu\nu} W^{A,\mu\nu}$

$\therefore\ e,\ g_Z,\ \sin^2\theta_Z$ = function of $g, g', C_i^{(n)}$
coefficients

‘Universal effect’, since all occurrences of $e,\ g_Z,\ \sin^2\theta_Z$ now carry
coefficient dependence

What can we do with this? `EW inputs'

With geoSMEFT setup, can set EW inputs to all orders:

$$e,\ g_Z,\ \sin^2\theta_Z \longrightarrow \text{functions of } g, g', h_{IJ}, g_{AB}$$

$$\begin{aligned}
\bar{g}_2 &= g_2\sqrt{g}^{11} = g_2\sqrt{g}^{22}, \\
\bar{g}_Z &= \frac{g_2}{c^2_{\theta_Z}}\left(c_{\bar\theta}\sqrt{g}^{33} - s_{\bar\theta}\sqrt{g}^{34}\right) = \frac{g_1}{s^2_{\theta_Z}}\left(s_{\bar\theta}\sqrt{g}^{44} - c_{\bar\theta}\sqrt{g}^{34}\right), \\
\bar{e} &= g_2\left(s_{\bar\theta}\sqrt{g}^{33} + c_{\bar\theta}\sqrt{g}^{34}\right) = g_1\left(c_{\bar\theta}\sqrt{g}^{44} + s_{\bar\theta}\sqrt{g}^{34}\right),
\end{aligned}$$

couplings

mixing angles

$$s^2_{\theta_Z} = \frac{g_1(\sqrt{g}^{44}s_{\bar\theta} - \sqrt{g}^{34}c_{\bar\theta})}{g_2(\sqrt{g}^{33}c_{\bar\theta} - \sqrt{g}^{34}s_{\bar\theta}) + g_1(\sqrt{g}^{44}s_{\bar\theta} - \sqrt{g}^{34}c_{\bar\theta})},$$

$$s^2_{\bar\theta} = \frac{(g_1\sqrt{g}^{44} - g_2\sqrt{g}^{34})^2}{g_1^2[(\sqrt{g}^{34})^2 + (\sqrt{g}^{44})^2] + g_2^2[(\sqrt{g}^{33})^2 + (\sqrt{g}^{34})^2] - 2g_1g_2\sqrt{g}^{34}(\sqrt{g}^{33} + \sqrt{g}^{44})}.$$

$$\bar{m}_W^2 = \frac{\bar{g}_2^2}{4}\sqrt{h_{11}}^2\bar{v}_T^2, \qquad \bar{m}_Z^2 = \frac{\bar{g}_Z^2}{4}\sqrt{h_{33}}^2\bar{v}_T^2 \qquad \bar{m}_A^2 = 0.$$

masses

[Helset, Martin, Trott 2001.01453]

Can get 'all orders' expressions for $1 \to 2$ processes:

e.g) $h \to \gamma\gamma$

go to mass basis

$$\langle h A^{\mu\nu} A_{\mu\nu} \rangle \mathcal{A}_{\rm SM}^{h\gamma\gamma} - \langle h A^{\mu\nu} A_{\mu\nu} \rangle \frac{\sqrt{h}^{44}}{4} \left[\langle \frac{\delta g_{33}(\phi)}{\delta \phi_4} \rangle \frac{\overline{e}^2}{g_2^2} + 2 \langle \frac{\delta g_{34}(\phi)}{\delta \phi_4} \rangle \frac{\overline{e}^2}{g_1 g_2} + \langle \frac{\delta g_{44}(\phi)}{\delta \phi_4} \rangle \frac{\overline{e}^2}{g_1^2} \right]$$

H normalization

expand $g_{33}(\phi) \mathscr{W}^3_{\mu\nu} \mathscr{W}^{3\mu\nu}$ to get linear h piece

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H normalization

expand $g_{33}(\phi)\mathscr{W}^3_{\mu\nu}\mathscr{W}^{3\mu\nu}$ to get linear h piece

application: expanding, can now calculate full $1/\Lambda^4$ corrections.
With that, we can:

- check how well (dim-6)^2 captures the effect
- treat $1/\Lambda^4$ as uncertainty and feed into fits on dim-6 coefficients
- think about how to pin down new coefficients with future measurements

Can get 'all orders' expressions for $1 \to 2$ processes:

e.g) $h \to \gamma\gamma$

defining: $$\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} = \left[\frac{g_2^2 \tilde{C}^{(6)}_{HB} + g_1^2 \tilde{C}^{(6)}_{HW} - g_1 g_2 \tilde{C}^{(6)}_{HWB}}{\left(g_1^2 + g_2^2\right) \bar{v}_T}\right]$$

(dim-6)² estimate: $$\left|\mathscr{A}^{h\gamma\gamma}_{SM}\right|^2 + 2\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} + \langle h|\gamma\gamma\rangle^2_{\mathscr{L}^{(6)}}$$

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Full $\mathscr{O}(1/\Lambda^4)$ result:

$$\left|\mathscr{A}_{SM}^{h\gamma\gamma}\right|^2 + 2\,\mathrm{Re}\left(\mathscr{A}_{SM}^{h\gamma\gamma}\right)\left(1 + \left\langle\sqrt{h}^{44}\right\rangle_{\mathscr{L}^{(6)}}\right)\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} + \left(1 + 4\bar{v}_T\,\mathrm{Re}\left(\mathscr{A}_{SM}^{h\gamma\gamma}\right)\right)\left(\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}}\right)^2$$
$$+ 2\,\mathrm{Re}\left(\mathscr{A}_{SM}^{h\gamma\gamma}\right)\left[\frac{g_2^2\tilde{C}_{HB}^{(8)} + g_1^2\left(\tilde{C}_{HW}^{(8)} - \tilde{C}_{HW,2}^{(8)}\right) - g_1 g_2 \tilde{C}_{HWB}^{(8)}}{\left(g_1^2 + g_2^2\right)\bar{v}_T}\right]$$

Can get 'all orders' expressions for $1 \to 2$ processes:

e.g) $h \to \gamma\gamma$

At $1/\Lambda^4$, only involves $\mathcal{O}(10)$ operators

Significant differences between full and (dim6)² result!

…even $\left(\langle h|\gamma\gamma\rangle_{\mathcal{L}^{(6)}}\right)^2$ captured incorrectly by just (dim-6)²

Full $\mathcal{O}(1/\Lambda^4)$ result:

$$\left|\mathcal{A}^{h\gamma\gamma}_{SM}\right|^2 + 2\,\mathrm{Re}\left(\mathcal{A}^{h\gamma\gamma}_{SM}\right)\left(1 + \left\langle\sqrt{h}^{44}\right\rangle_{\mathcal{L}^{(6)}}\right)\langle h|\gamma\gamma\rangle_{\mathcal{L}^{(6)}} + \left(1 + 4\bar{v}_T\,\mathrm{Re}\left(\mathcal{A}^{h\gamma\gamma}_{SM}\right)\right)\left(\langle h|\gamma\gamma\rangle_{\mathcal{L}^{(6)}}\right)^2$$
$$+2\,\mathrm{Re}\left(\mathcal{A}^{h\gamma\gamma}_{SM}\right)\left[\frac{g_2^2\tilde{C}^{(8)}_{HB} + g_1^2\left(\tilde{C}^{(8)}_{HW} - \tilde{C}^{(8)}_{HW,2}\right) - g_1 g_2 \tilde{C}^{(8)}_{HWB}}{\left(g_1^2 + g_2^2\right)\bar{v}_T}\right]$$

Working to $1/\Lambda^4$: bottom up

e.g) $h \to \gamma\gamma$

Quantify effect by randomly drawing coefficients and comparing dim-6, (dim-6)^2 and full $1/\Lambda^4$ result

How do you randomly draw coefficients?

For weakly coupled UV theories, well known classification of operators up to dim-8 into 'tree' and 'loop' level

[Arzt'93], [Einhorn, Wudka '13], [Craig et al '20]

	Tree	Loop
Dim 6	$\bar{\psi}\psi H^2 D,\ H^4 D^2, \psi^2 H^3, \cdots$	$H^2 X^2,\ X^3, \cdots$
Dim 8	$H^4 X^2,\ \psi^4 X,\ \psi^5 H\ \cdots$ $\bar{\psi}\psi H^4 D$	$H^2 X^3,\ X^4, \cdots$

$\psi, \bar{\psi}$ = any fermion, H = Higgs, X = any field strength, D = covariant derivative

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Dim 8	$H^4 X^2,\ \psi^4 X,\ \psi^5 H\ \cdots$ $\bar{\psi}\psi H^4 D$	$H^2 X^3,\ X^4, \cdots$ impact $h \to \gamma\gamma, h \to Z\gamma$

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	Tree	Loop
Dim 6	$\bar{\psi}\psi H^2 D$, $H^4 D^2, \psi^2 H^3, \cdots$	$H^2 X^2$, $X^3, \cdots$
Dim 8	$H^4 X^2$, $\psi^4 X$, $\psi^5 H$ $\cdots$ $\bar{\psi}\psi H^4 D$	$H^2 X^3$, $X^4, \cdots$ impact $h \to \gamma\gamma, h \to Z\gamma$ impact $Z \to \bar{\psi}\psi$

$\psi, \bar{\psi}$ = any fermion, H = Higgs, X = any field strength, D = covariant derivative

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e.g) $h \to \gamma\gamma$

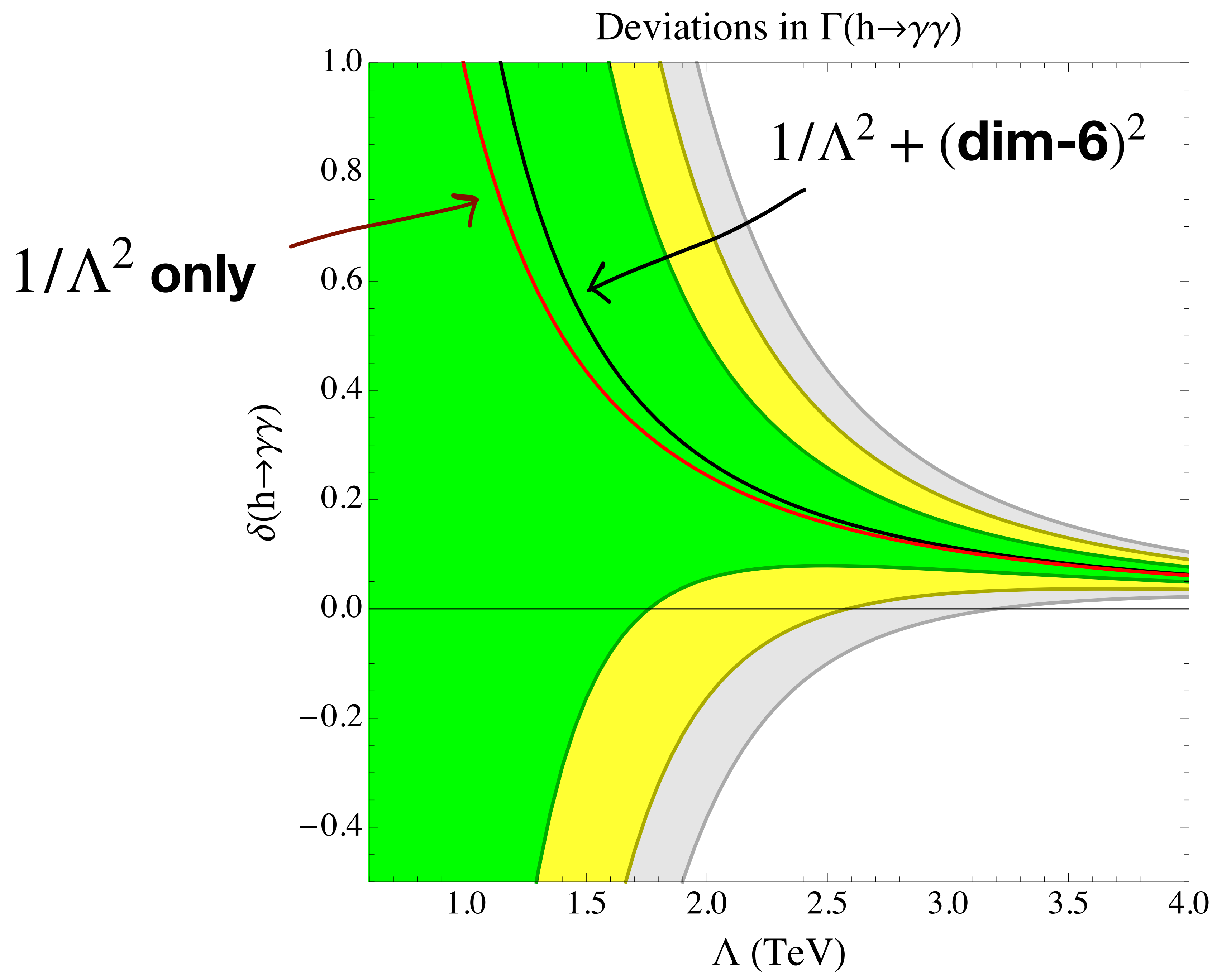
Quantify effect by randomly drawing coefficients and comparing dim-6, (dim-6)^2 and full $1/\Lambda^4$ result

Tree level operators: draw coefficients at random from a gaussian with **mean 0, width 1**

Loop level operators: draw coefficients at random from a gaussian with **mean 0, width 0.01**

Working to $1/\Lambda^4$: bottom up

e.g) $h \to \gamma\gamma$



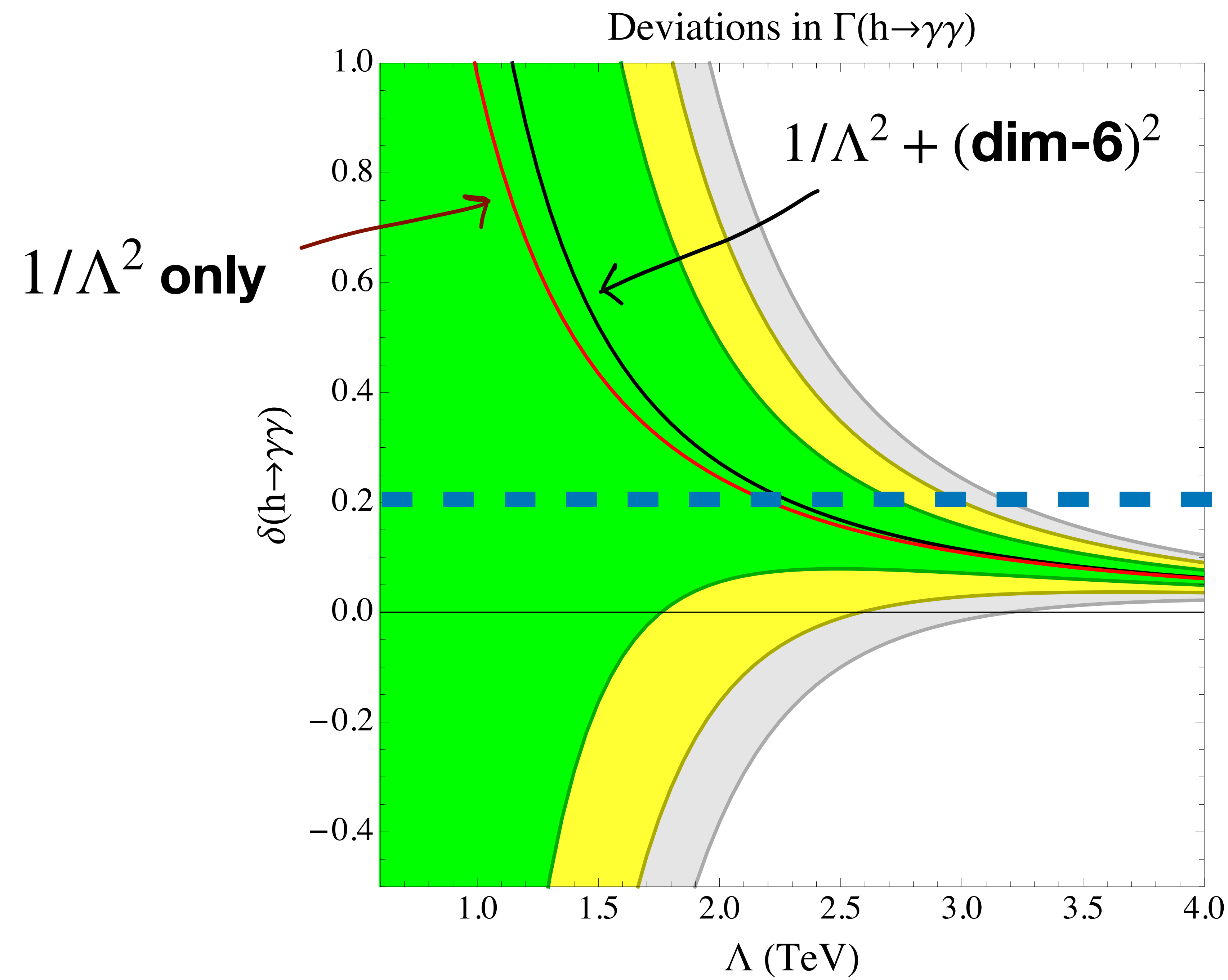
Contours show range of effects once full $1/\Lambda^4$ effects are included (for fixed $1/\Lambda^2$, (dim-6)^2 result)

Working to $1/\Lambda^4$: bottom up

e.g) $h \to \gamma\gamma$

For fixed deviation, e.g.

$$\delta(h \to \gamma\gamma) = 0.2$$



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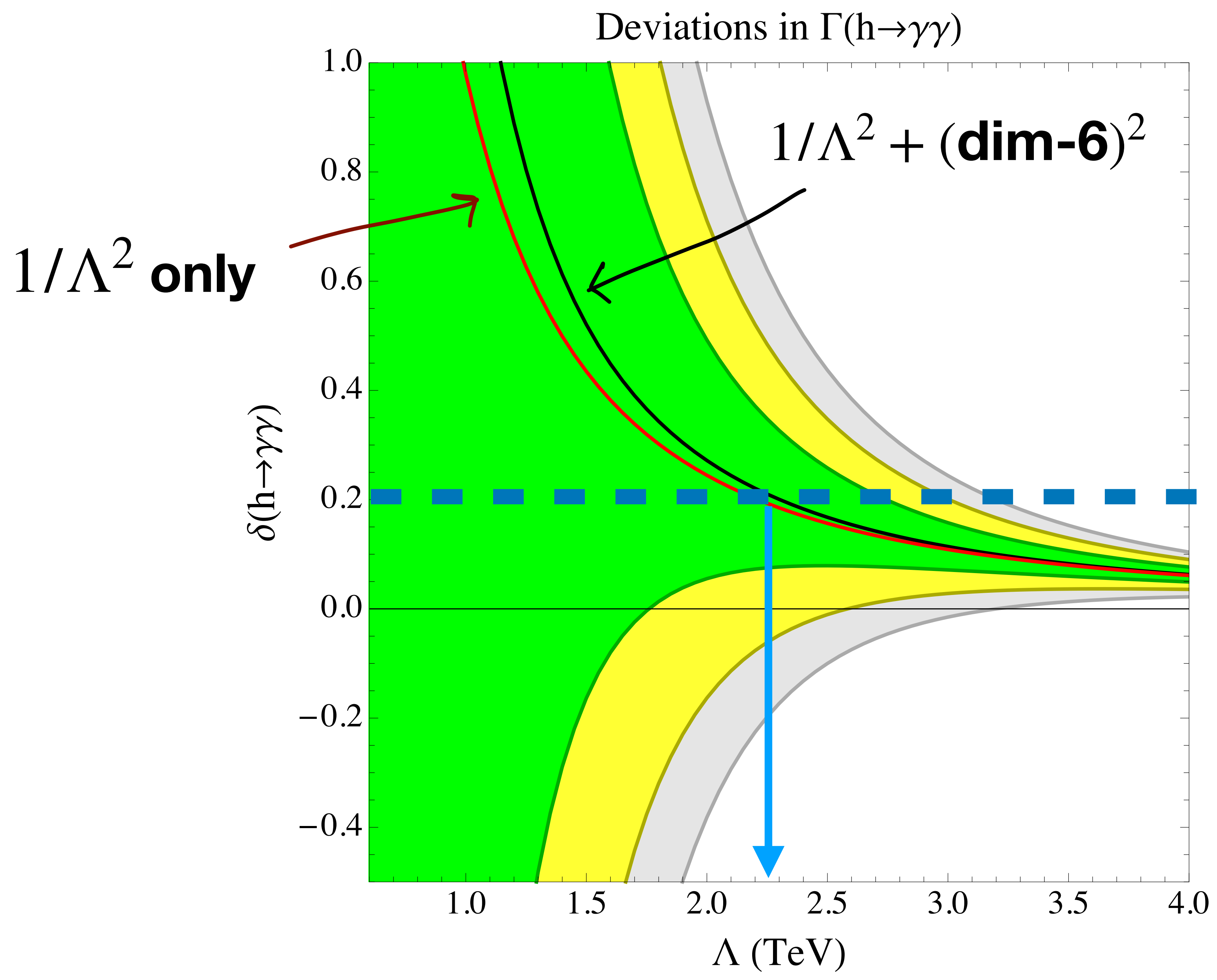

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Λ interpretation assuming interference only: ~2.3 TeV

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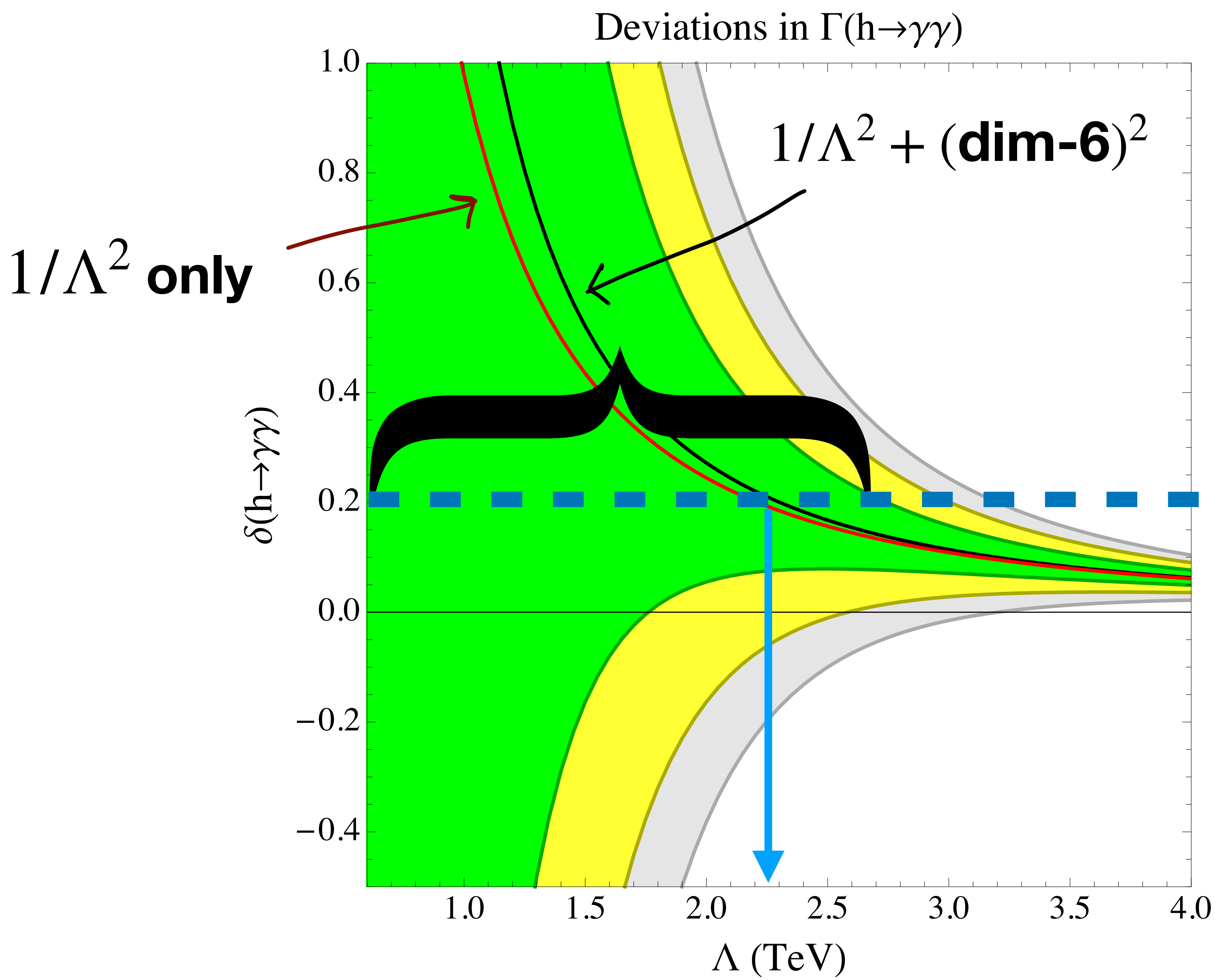


For fixed deviation, e.g.

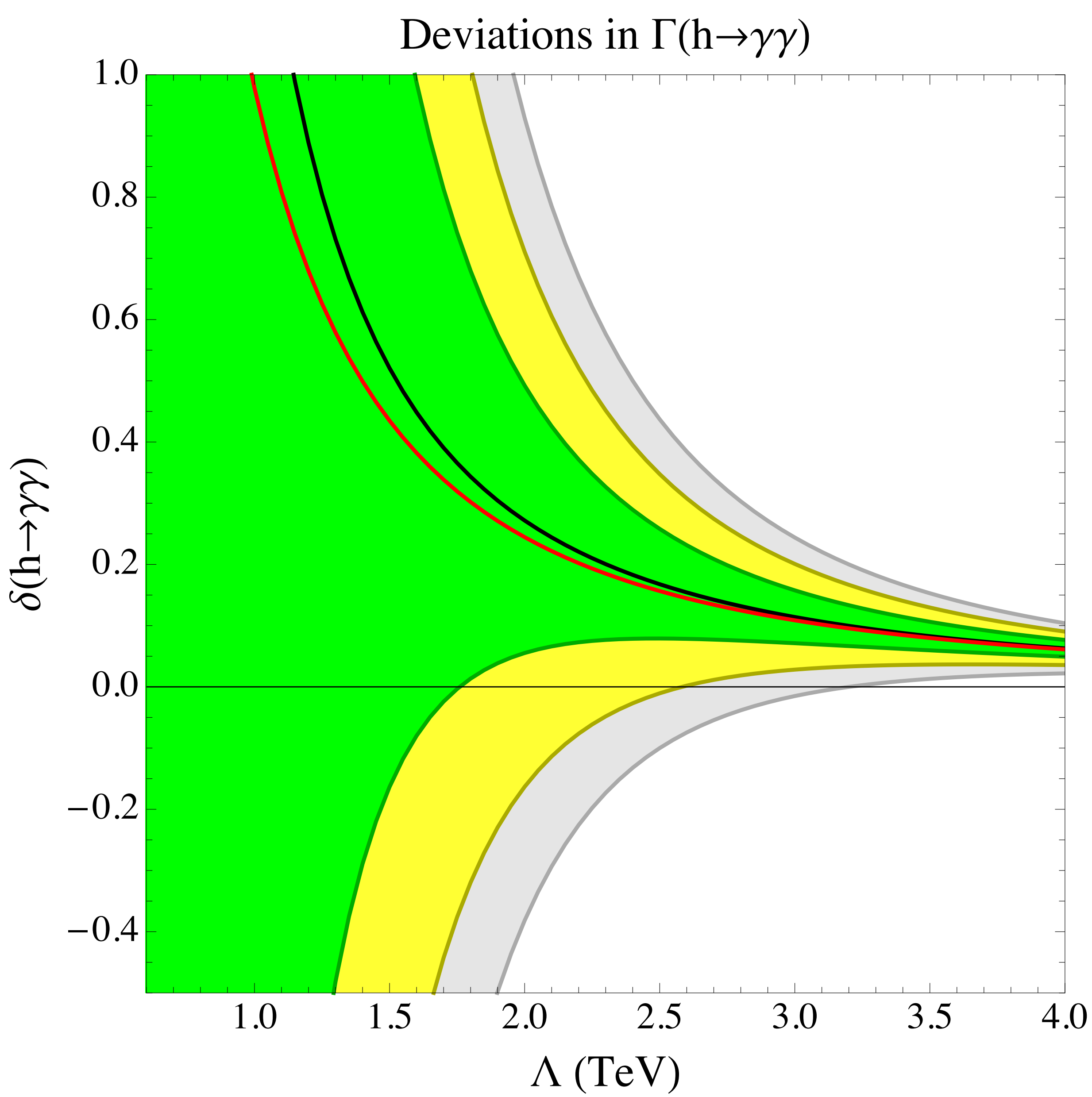
$$\delta(h \to \gamma\gamma) = 0.2$$

Λ interpretation assuming interference only: ~2.3 TeV

Λ interpretation with full : [0.5 - 2.7 TeV]

Working to $1/\Lambda^4$: bottom up

e.g) $h \to \gamma\gamma$



Why such a large effect?

Following tree/loop classification, all operators at dim-6 are loop-level

$$\langle h|\gamma\gamma\rangle_{to\, v^2/\Lambda^2} \sim 0.01\left(\frac{C^{(6)}}{0.01}\right)\frac{v^2}{\Lambda^2}$$

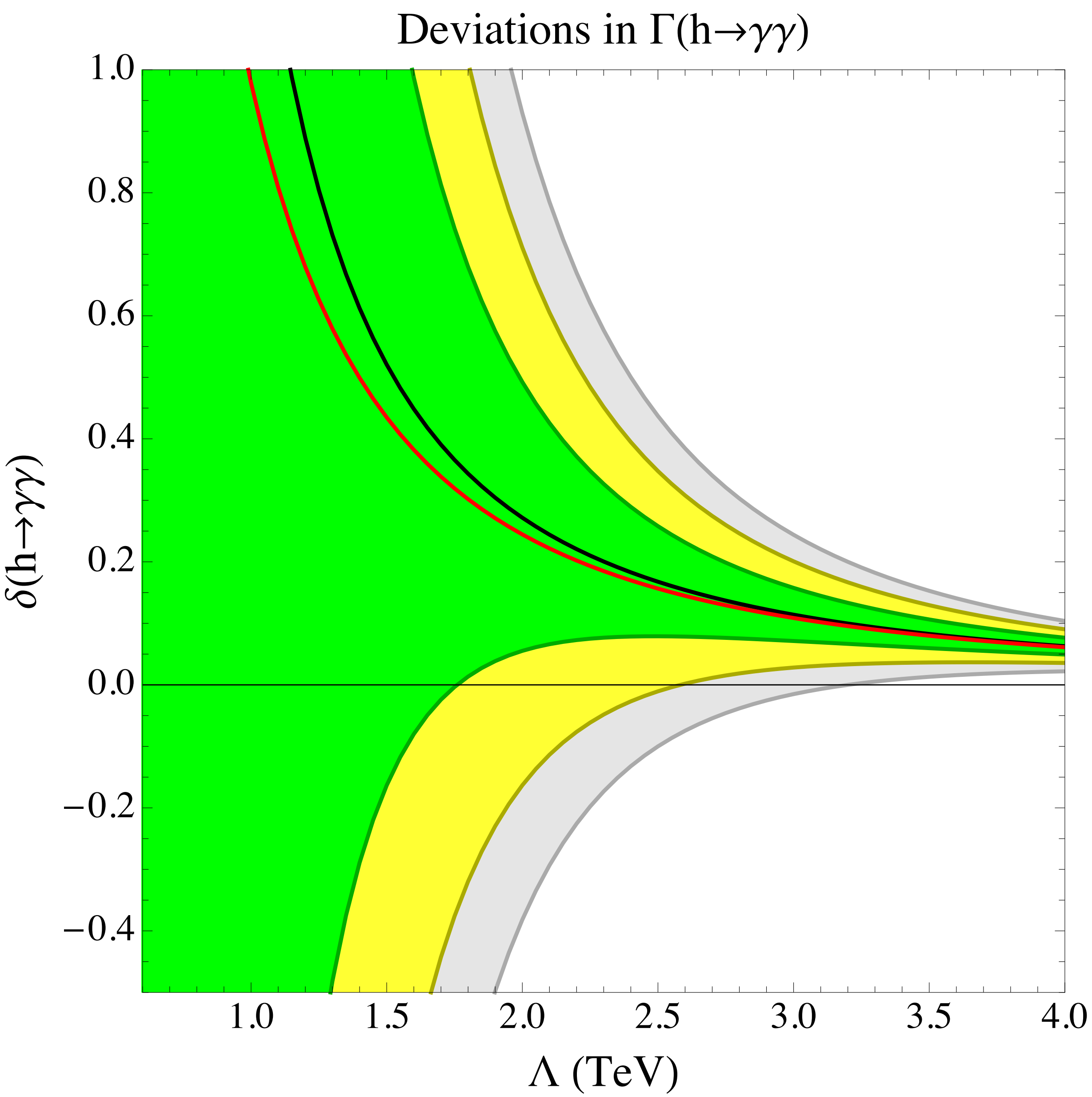
Tree effects enter at dim-8

$$\langle h|\gamma\gamma\rangle_{to\, v^4/\Lambda^4} \sim \left(\frac{C^{(8)}}{1.0}\right)\frac{v^4}{\Lambda^4}$$

similar result for $h \to Z\gamma$

Working to $1/\Lambda^4$: bottom up

e.g) $h \to \gamma\gamma$



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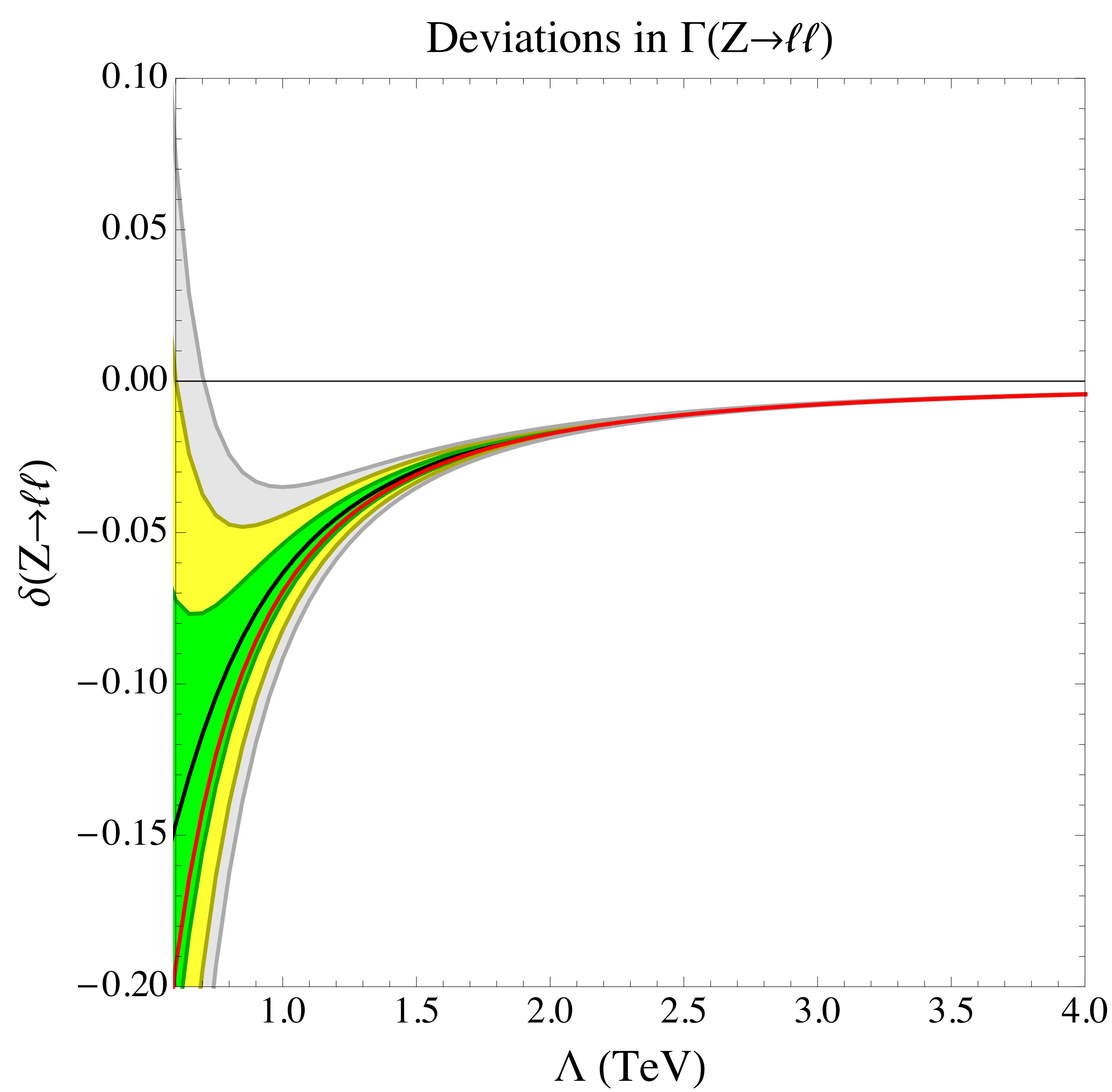
effects can compete despite higher order in Λ

similar result for $h \to Z\gamma$

Working to $1/\Lambda^4$: bottom up

e.g.) $Z \to \ell^+\ell^-$



Now tree-level operators present for both dim-6 and dim-8

$$\langle Z|\ell\ell\rangle_{to\, v^2/\Lambda^2} \sim \left(\frac{C^{(6)}}{1.0}\right)\frac{v^2}{\Lambda^2}$$

$$\langle Z|\ell\ell\rangle_{to\, v^4/\Lambda^4} \sim \left(\frac{C^{(8)}}{1.0}\right)\frac{v^4}{\Lambda^4}$$

smaller impact, but still present, especially if Λ is small

Working to $1/\Lambda^4$: top down

Try a specific UV model: kinetically mixed U(1)

$$\Delta\mathscr{L} = -\frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{1}{2}m_K^2 K_\mu K^\mu - \frac{k}{2}B^{\mu\nu}K_{\mu\nu}$$

integrate out to dim-8 (tree level only)

$$\Delta\mathscr{L} = -\frac{k^2}{2m_K^2}j_\mu j^\mu + \frac{k^2 - k^4}{2m_K^4}\left(\partial^2 j_\mu\right)j^\mu + \frac{g_1^2 k^4}{4m_K^4}\left(H^\dagger H\right)j_\mu j^\mu$$

where

$$j_\mu = \sum_\psi \left(-g_1 \mathbf{y}_\psi\right)\bar{\psi}\gamma_\mu\psi + \left(-\frac{1}{2}g_1\right)H^\dagger i D_\mu H$$

Working to $1/\Lambda^4$: top down

dim-6

$H^2\psi^2 D$

$C_{H\ell}^{1,(6)}$	$-\frac{y_\ell g_1^2}{2m_K^2} b_1$
$C_{He}^{(6)}$	$-\frac{y_e g_1^2}{2m_K^2} b_1$
$C_{Hq}^{1,(6)}$	$-\frac{y_q g_1^2}{2m_K^2} b_1$
$C_{Hu}^{(6)}$	$-\frac{y_u g_1^2}{2m_K^2} b_1$
$C_{Hd}^{(6)}$	$-\frac{y_d g_1^2}{2m_K^2} b_1$

...

$H^4 D^2$

$C_{H\Box}^{(6)}$	$-\frac{g_1^2 k^2}{8m_K^2}$
$C_{HD}^{(6)}$	$-\frac{g_1^2 k^2}{2m_K^2}$

$\psi^4 : (\bar{L}L)(\bar{L}L)$

$C_{\ell\ell}^{(6)}$	$-\frac{1}{8}\frac{g_1^2 k^2}{m_K^2}$
$C_{qq}^{1,(6)}$	$-\frac{1}{72}\frac{g_1^2 k^2}{m_K^2}$
$C_{\ell q}^{1,(6)}$	$\frac{1}{12}\frac{g_1^2 k^2}{m_K^2}$

dim-8

$H^4\psi^2 D$

$C_{H\ell}^{1,(8)}$	$\frac{y_\ell g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_\ell}{m_K^4}(k^2-k^4)(2\lambda + \frac{g_1^2+g_2^2}{4})$
$C_{He}^{1,(8)}$	$\frac{y_e g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_e}{m_K^4}(k^2-k^4)(2\lambda + \frac{g_1^2+g_2^2}{4})$
$C_{Hq}^{1,(8)}$	$\frac{y_q g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_q}{m_K^4}(k^2-k^4)(2\lambda + \frac{g_1^2+g_2^2}{4})$
$C_{Hu}^{1,(8)}$	$\frac{y_u g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_u}{m_K^4}(k^2-k^4)(2\lambda + \frac{g_1^2+g_2^2}{4})$
$C_{Hd}^{1,(8)}$	$\frac{y_d g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_d}{m_K^4}(k^2-k^4)(2\lambda + \frac{g_1^2+g_2^2}{4})$
$C_{H\ell}^{2,(8)}$	$-\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$
$C_{Hq}^{2,(8)}$	$-\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$
$C_{H\ell}^{3,(8)}$	$-\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$
$C_{Hq}^{3,(8)}$	$-\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$

$H^6 D^2$

$C_{H,D2}^{(8)}$	$\frac{g_1^4\,k^4}{8\,m_K^4} - \frac{g_1^2\,g_2^2}{2\,m_K^4}(k^2-k^4)$
$C_{HD}^{(8)}$	$\frac{3\,g_1^4\,k^4}{16\,m_K^4} - \frac{g_1^2\,g_2^2}{2\,m_K^4}(k^2-k^4)$

$X^2 H^4$

$C_{HB}^{(8)}$	$-\frac{g_1^4}{16\,m_K^4}(k^2-k^4)$
$C_{HW}^{(8)}$	$\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$

No operators that impact $h \to \gamma\gamma$

operators impacting $h \to \gamma\gamma$ present

$\therefore$ at dim-6 level, no effect, while there is an effect if we go to full $1/\Lambda^4$

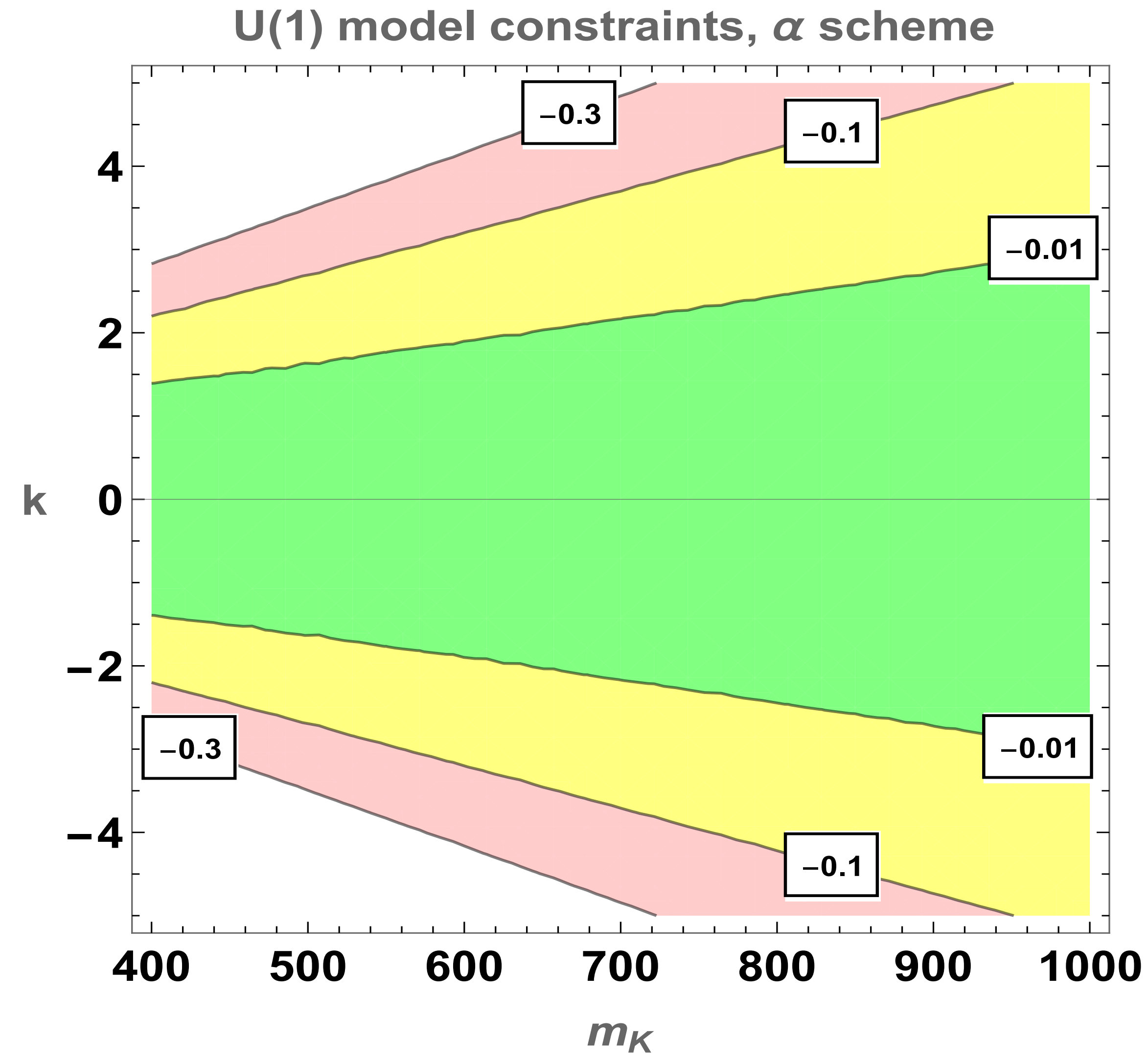
Working to $1/\Lambda^4$: top down

Said differently:

If restricted to $1/\Lambda^2$ level, appears like no constraint from $h \to \gamma\gamma$

But done fully at $1/\Lambda^4$, constraint is there

$$\frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{\text{SMEFT}}}$$



So where does this leave us?

Restricted to 2- and 3-pt resonant phenomenology, can think about $1/\Lambda^4$ effects (and beyond!) without introducing a flood of new operators

- geoSMEFT framework: basis where 2 and 3 particle vertices sensitive to a minimal # of operators, # ~ constant with mass dimension

- Can study select processes to $1/\Lambda^4$, use them to form guidelines for how to include truncation error more generally in SMEFT studies

Find (dim-6)^2 is not a great proxy for full $1/\Lambda^4$ effects, especially for loop-level SM processes

So where does this leave us?

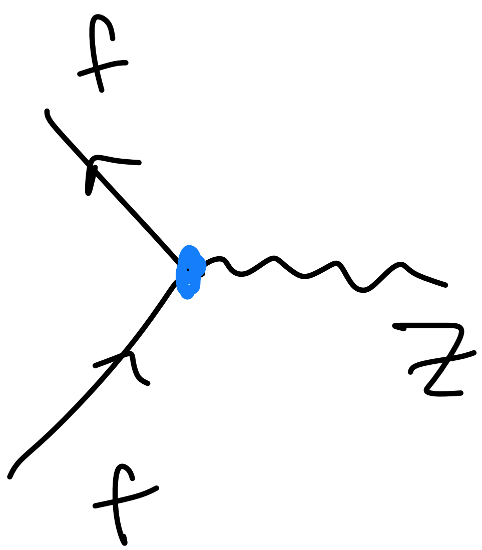
Lots to do:

- Expand the 'laboratory': more $1 \to 2,\ 2 \to 2$ processes
- Incorporate into dim-6 coefficient fits
- Combine with effects from higher loop (NLO) order
- How to pin down new coefficients, rather than treat them as nuisance parameters?

THANK YOU!

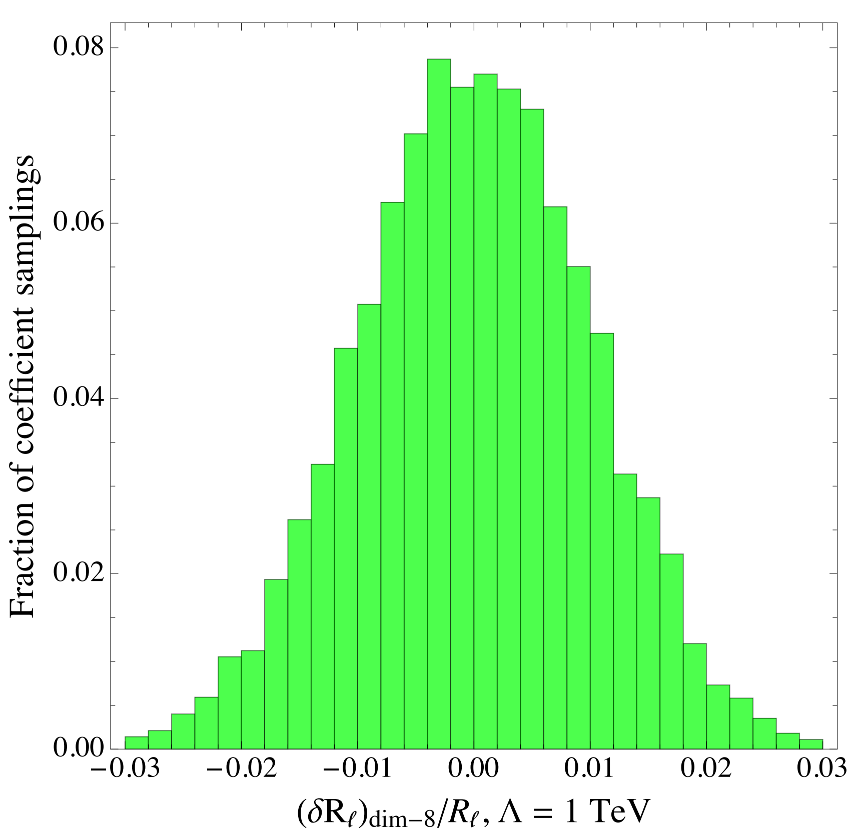
Backup

Redo classic SMEFT LEP1 analysis to $\mathcal{O}(1/\Lambda^4)$



$$g^{\mathcal{Z},\psi}_{\text{eff,pr}} = \frac{\bar{g}_Z}{2}\left[(2s^2_{\theta_Z}Q_\psi - \sigma_3)\delta_{pr} + \bar{v}_T\langle L^{\psi,pr}_{3,4}\rangle + \sigma_3\bar{v}_T\langle L^{\psi,pr}_{3,3}\rangle\right]$$
$$= \langle g^{\mathcal{Z},\psi}_{\text{SM,pr}}\rangle + \langle g^{\mathcal{Z},\psi}_{\text{eff,pr}}\rangle_{\mathcal{O}(v^2/\Lambda^2)} + \langle g^{\mathcal{Z},\psi}_{\text{eff,pr}}\rangle_{\mathcal{O}(v^4/\Lambda^4)} + \cdots .$$

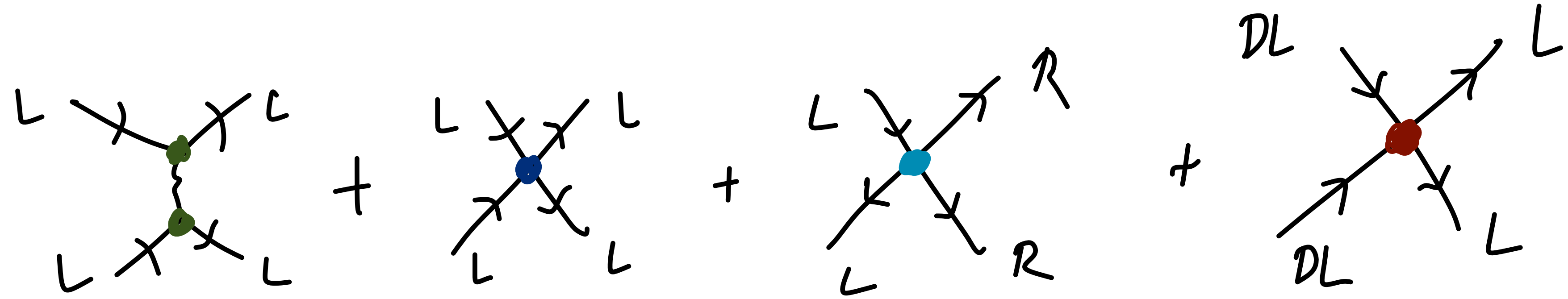
scanning dim-8 coefficients



SMEFT corrections in $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}/\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ scheme			
$\mathcal{O}(\frac{v^4}{\Lambda^4})$	$\langle g^{\mathcal{Z},u_R}_{\text{eff,pp}}\rangle$	$\langle g^{\mathcal{Z},d_R}_{\text{eff,pp}}\rangle$	$\langle g^{\mathcal{Z},\ell_R}_{\text{eff,pp}}\rangle$
$\langle g^{\mathcal{Z},\psi}_{\text{eff}}\rangle^2$	14/5.5	-27/-11	-9.1/-3.6
$\tilde{C}_{HB}\,C_{HWB}$	-0.21/0.39	0.10/-0.19	0.31/-0.58
$\tilde{C}^2_{HD}$	0.28/-0.026	-0.14/0.013	-0.42/0.040
$\tilde{C}_{HD}\,\tilde{C}^{(6)}_{H\psi}$	-0.83/-0.19	-0.83/-0.19	-0.83/-0.19
$\tilde{C}_{HD}\,\tilde{C}_{HWB}$	0.59/-0.19	-0.29/0.097	-0.88/0.29
$\tilde{C}_{HD}\langle g^{\mathcal{Z},\psi}_{\text{eff}}\rangle$	4.0/0.50	4.0/0.50	4.0/0.50
$(\tilde{C}^{(6)}_{H\psi})^2$	0.62/1.4	-1.2/-2.8	-0.42/-0.93
$\tilde{C}_{HWB}\,\tilde{C}^{(6)}_{H\psi}$	-0.69/0.58	-0.69/0.58	-0.69/0.58
$\tilde{C}^{(6)}_{H\psi}\langle g^{\mathcal{Z},\psi}_{\text{eff}}\rangle$	-6.7/-5.8	13/12	4.5/3.9
$\tilde{C}_{HWB}\,\langle g^{\mathcal{Z},\psi}_{\text{eff}}\rangle$	3.7/0.26	3.7/0.26	3.7/0.26
$\tilde{C}_{HW}\,C_{HWB}$	-0.21/0.39	0.10/-0.19	0.31/-0.58
$\tilde{C}^{(8)}_{HD}$	-0.014/0.026	0.0069/-0.013	0.021/-0.040
$\tilde{C}^{(8)}_{HD,2}$	-0.21/0.026	0.10/-0.013	0.31/-0.040
$\tilde{C}^{(8)}_{H\psi}$	0.19/0.19	0.19/0.19	0.19/0.19
$\tilde{C}^{(8)}_{HW,2}$	-0.38/0	0.19/0	0.58/0
$\tilde{C}^{(8)}_{HWB}$	-0.10/0.19	0.051/-0.097	0.15/-0.29
$\delta G^{(8)}$	-0.078/0.15	0.039/-0.075	0.12/-0.22

What about G_F?

G_F involves more than quadratic terms:



However, since G_F derived at muon mass scale $(D \sim m_\mu \ll \Lambda)$ and SM term is from L^4, # of higher dimensional contributions is dramatically reduced

$$C^{(8+2n)}_{4\ell,2} \left(H^\dagger H\right)^{1+n} \left(\bar{\ell}_2 \gamma^\mu \sigma^i \ell_2\right) \left(\bar{\ell}_1 \gamma_\mu \sigma_i \ell_1\right) \qquad i C^{(8+2n)}_{4\ell,5} \epsilon_{ijk} \left(H^\dagger H\right)^n \left(H^\dagger \sigma^i H\right) \left(\bar{\ell}_2 \gamma^\mu \sigma_j \ell_2\right) \left(\bar{\ell}_1 \gamma_\mu \sigma_k \ell_1\right)$$

All orders result is possible even for contact terms:

$$\mathscr{G}_F^{4pt} = \frac{1}{\bar{v}_T^2} \left(\tilde{C}^{(6)}_{\mu c c \mu} + \tilde{C}^{(6)}_{\mu\mu\mu e} + \frac{\tilde{C}^{(8+2n)}_{4\ell,2}}{2^n} + \frac{\tilde{C}^{(8+2n)}_{4\ell,5}}{2^n} \right)$$

[Hays, Helset, Martin, Trott 2007.00565]

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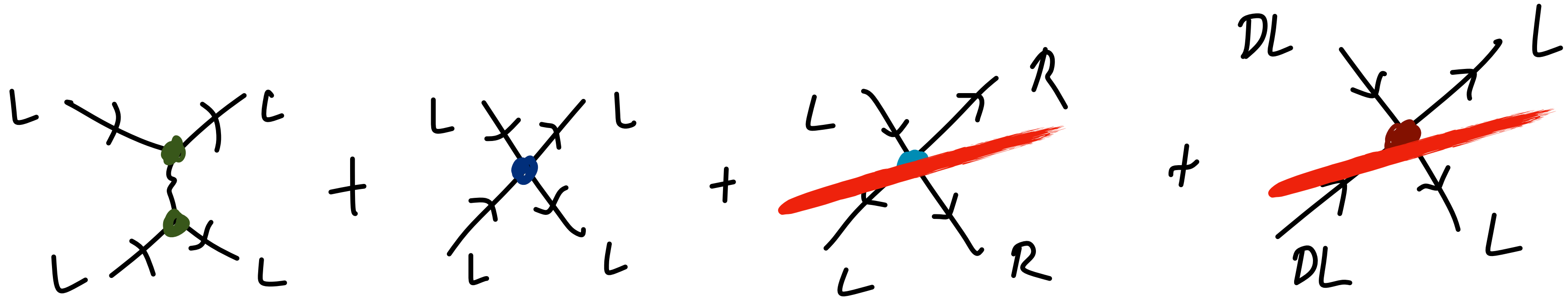

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[Hays, Helset, Martin, Trott 2007.00565]

Tree vs. Loop

[from Craig et al '20]

$$\mathscr{L}_{\mathrm{UV}} = \frac{1}{2}\Omega^T K \Omega - \Omega^T J + \mathscr{O}\left(\Omega^3\right)$$

$$\Omega = \begin{pmatrix} \Phi \\ \Psi \\ \bar{\Psi} \\ V_\mu \end{pmatrix} \qquad K = \begin{pmatrix} -D^2 - M^2 & -y\psi & -y\bar{\psi} & 0 \\ -y\psi & -M - y\phi & -(\bar{\sigma}\cdot iD)^T & 0 \\ -y\bar{\psi} & \bar{\sigma}\cdot iD & -M - y\phi & 0 \\ 0 & 0 & 0 & \eta^{\mu\nu}\left(D^2 + M^2 + g\phi^2\right) - D^\nu D^\mu + [D^\mu, D^\nu] \end{pmatrix} \qquad J = \begin{pmatrix} y\psi\psi + y\bar{\psi}\bar{\psi} + \lambda\phi^3 \\ y\phi\psi \\ y\phi\bar{\psi} \\ g\bar{\psi}\sigma^\mu\psi + g\phi \overleftrightarrow{D}^\mu \phi \end{pmatrix}$$

heavy stuff

mass of heavy stuff **M**,
interactions with 2 heavy fields

light stuff

Integrate out Ω at tree-level = solve EOM, expand in **1/M**

$$\mathcal{L}_{\mathrm{EFT}} \supset \frac{1}{2M^2} J^T \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & M - y\phi & -(\bar{\sigma}\cdot iD)^T & 0 \\ 0 & \bar{\sigma}\cdot iD & M - y\phi & 0 \\ 0 & 0 & 0 & -\eta^{\mu\nu} \end{pmatrix} J \quad \mathbf{+ \ldots}$$

(at dim-6, similar but lengthier for dim-8)

Expanding out, can see what terms are present.
E.g.) no field strengths at dim-6!

Powerful new tool

of operators and their field content can be generated automatically via **Hilbert Series**

[Lehman, AM ’15, Henning et al ’15, ’17]

given symmetry group **G**, fields ϕ_i, ψ_i, $X_i^{L,R}$ → Hilbert Series → # and form of **all** invariant (Lorentz & gauge) operators,

Powerful new tool

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[Lehman, AM ’15, Henning et al ’15, ’17]

given symmetry group **G**, fields $\boldsymbol{\phi_i}$, $\boldsymbol{\psi_i}$, $\boldsymbol{X_i^{L,R}}$ → **Hilbert Series** → # and form of **all** invariant (Lorentz & gauge) operators,

- extends to all orders
- includes all IBP, EOM redundancies
- works for all sorts of EFT (SMEFT, nonlinear reps, non-relativistic QFT)

[Kobach, Pal ’17, ’18, Graf et al ’20]

Hilbert series:

$$\mathcal{H}_{SM} = \int d\mu_{Lorentz}\, d\mu_{gauge} \frac{1}{P} PE\Big[\sum_{\phi} \frac{\phi}{\mathcal{D}^{d_\phi}} \chi_\phi\Big] PEF\Big[\sum_{\psi} \frac{\psi}{\mathcal{D}^{d_\psi}} \chi_\psi\Big]$$

projects out invariants from polynomial (relies on character orthonormality)

generating function — generates all possible polynomials of fields (ϕ^2, $\phi\, \psi$, $\psi^2\, \phi$,etc.) and derivatives

removes IBP redundancies

Real representation translation

Using γ_A = generators in real representation and $\Gamma_A = \gamma_A \gamma_4$, translate

$$H^\dagger \sigma_a H = -\frac{1}{2}\phi_I \Gamma^I_{a,J}\phi^J$$

$$H^\dagger \hat{D}^\mu H = -\phi_I \gamma^I_{4,J}\left(D^\mu \phi\right)^J = \left(D^\mu \phi\right)_I \gamma^I_{4,J}\phi^J$$

$$H^\dagger \hat{D}\hat{D}^\mu_a H = -\phi_I \gamma^I_{a,J}\left(D^\mu \phi\right)^J = \left(D^\mu \phi\right)_I \gamma^I_{a,J}\phi^J$$