PROBLEMS 26 SCORES INSIGHTS SETTINGS

+ Add content

Differential Equations Fall 2022

Home

Instructor Courses 3

Course Catalog

Problem Repository

Help Center

Preview Find Reorder Set points

1 Check all that apply to the differential equation $\frac{d^2y}{dx^2} + y = 0$

+ Add content

Differential Equations Fall 2022
Home
Instructor Courses
Course Catalog
Problem Repository
Help Center

- **A.** ODE
- **B.** PDE
- **C.** first order
- **D.** second order
- **E.** third order
- **F.** linear
- **G.** a candidate for integrating factors
- **H.** separable
- **I.** homogeneous
- **J.** autonomous

2 Check all that apply to the differential equation $\frac{d^3y}{dx^3} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x$

- **A.** ODE
- **B.** PDE
- **C.** first order
- **D.** second order
- **E.** third order
- **F.** linear
- **G.** a candidate for integrating factors
- **H.** separable
- **I.** homogeneous
- **J.** autonomous

3 Check all that apply to the differential equation $P'' + 2P' = 0$

- **A.** ODE
- **B.** PDE
- **C.** first order
- **D.** second order
- **E.** third order
- **F.** linear
- **G.** a candidate for integrating factors
- **H.** separable
- **I.** homogeneous
- **J.** autonomous

4 Solve the differential equation by the method of integrating factors.

$$\frac{dy}{dx} + 2xy = 10x$$

Use "C" to represent any constant of integration.

+ Add content

Differential Equations Fall 2022

Home

Instructor Courses

Course Catalog

Problem Repository

Help Center

5 Solve $xy' = 9y - 1x, \quad y(1) = 0.$

(a) Identify the integrating factor, $\alpha(x)$.

$\alpha(x) =$

(b) Find the general solution.

$y(x) =$

Note: Use C for the arbitrary constant.

(c) Solve the initial value problem $y(1) = 0$.

$y(x) =$

6 Consider the logistic equation

$$\dot{y} = y\,(1 - y)$$

(a) Find the solution satisfying $y_1(0) = 16$ and $y_2(0) = -5$.

$y_1(t) =$

$y_2(t) =$

(b) Find the time t when $y_1(t) = 8$.

$t =$

(c) When does $y_2(t)$ become infinite?

$t =$

7 Solve the separable differential equation.

$$y' = 10y^2$$

Use the following initial condition: $y(10) = 2$

$y =$

Note: Your answer should be a function of x.

+ Add content

Differential Equations Fall 2022

Home

Instructor Courses

Course Catalog

Problem Repository

Help Center

8 (1 point) Use the "mixed partials" check to see if the following differential equation is exact. If it is exact find a function F(x,y) whose differential, $dF(x,y)$ is the left hand side of the differential equation. That is, level curves $F(x,y) = C$ are solutions to the differential equation:

$$(4xy^2 - 4y)dx + (4x^2y - 4x)dy = 0$$

First:

$M_y(x,y) =$ [ ] , and $N_x(x,y) =$ [ ] .

If the equation is not exact, enter *not exact,* otherwise enter in $F(x,y)$ here

[ ]

9 a. Find a particular solution to the nonhomogeneous differential equation $y'' + 4y' + 5y = 15x + e^{-x}$.

$y_p =$ [ ]

b. Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ [ ]

c. Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ [ ]

10 (1 point) Use the method of undetermined coefficients to solve the following differential equation:

$$y'' + y' = 4x$$

Answer: $y(x) =$ [ ] $+C_1$ [ ] $+C_2$ [ ] .

NOTE: The order of your answers is important in this problem. For example, webwork may expect the answer "A+B" but the answer you give is "B+A". Both answers are correct but webwork will only accept the former.

11 (1 point) Use the method of undetermined coefficients to solve the following differential

equation:

$$y'' + 6y' + 9y = 2\sin(x)$$

Answer: $y(x) =$ $+C_1$ $+C_2$

.

NOTE: The order of your answers is important in this problem. For example, webwork may expect the answer "A+B" but the answer you give is "B+A". Both answers are correct but webwork will only accept the former.

12 (1 point) Use the method of undetermined coefficients to solve the following differential equation:

$$y'' + 6y' + 9y = 2e^{-x}$$

Answer: $y(x) =$ $+C_1$ $+C_2$

.

NOTE: The order of your answers is important in this problem. For example, webwork may expect the answer "A+B" but the answer you give is "B+A". Both answers are correct but webwork will only accept the former.

13 It can be shown that $y_1 = e^{5x}$ and $y_2 = xe^{5x}$ are solutions to the differential equation $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0$.

$W(y_1, y_2)$ = .

$c_1y_1 + c_2y_2$ is the general solution to the equation on the interval

14 Find a particular solution to $y'' + 10y' + 25y = \dfrac{e^{-5x}}{x^3}$

$y_p =$

15 (1 point) Solve the following differential equation:

$$y'' + 4y = \sin^3 x$$

Answer: $y(x) =$ $+C_1$ $+C_2$

.

16 (1 point) Solve the initial value problem

$$(5+x^2)y'' + 1y = 0,\ y(0) = 0,\ y'(0) = 7.$$

If the solution is

$$y = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5 + c_6x^6 + c_7x^7 + \cdots,$$

enter the following coefficients:

$c_0 =$ 3

$c_1 =$

$c_2 =$

$c_3 =$

$c_4 =$

$c_5 =$

$c_6 =$

17 Assume that y is the solution of the initial-value problem

$$y' + 4y = \begin{cases} \frac{2\sin x}{x} & x \neq 0 \\ 2 & x = 0 \end{cases}, \qquad y(0) = 1\ .$$

If y is written as a power series

$$y = \sum_{n=0}^{\infty} c_n x^n\ ,$$

then the first few terms are

$$\square + \square\, x + \square\, x^2 + \square\, x^3 + \square\, x^4\ .$$

Note: You do not have to find a general expression for c_n. Just find the coefficients one by one.

18 The general solution of the differential equation $x^2y'' + xy' + (81x^2 - 4)y = 0$ is?

- **A.** $c_1J_2(9x) + c_2Y_2(9x)$
- **B.** $c_1J_{-9}(2x) + c_2Y_9(2x)$
- **C.** $c_1J_9(2x) + c_2Y_9(2x)$
- **D.** $c_1J_{-2}(9x) + c_2J_2(9x)$

Add content

Differential Equations Fall 2022

Home

Instructor Courses

Course Catalog

Problem Repository

Help Center

Set up an integral for finding the Laplace transform of the following function:

$$f(t) = \begin{cases} 0, & 0 \le t < 2 \\ t - 9, & 2 \le t. \end{cases}$$

$F(s) = \mathcal{L}\{f(t)\} = \int_A^B$ [ ]

where $A =$ [ ] and $B =$ [ ] .

b. Find the antiderivative (with constant term 0) corresponding to the previous part.

[ ]

c. Evaluate appropriate limits to compute the Laplace transform of $f(t)$:

$F(s) = \mathcal{L}\{f(t)\} =$ [ ]

d. Where does the Laplace transform you found exist? In other words, what is the domain of $F(s)$?

20 (1 point) Find the inverse Laplace transform of

$$\frac{9s + 2}{s^2 + 17} \qquad s > 0$$

$y(t) =$ [ ] .

21 Compute the inverse Laplace transform:

$$\mathcal{L}^{-1}\left\{\frac{-3}{s^2 - 5s + 4} e^{-4s}\right\} =$$ [ ]

(**Notation:** write **u(t-c)** for the Heaviside step function $u_c(t)$ with step at $t = c$.)

If you don't get this in 2 tries, you can get a hint.

22 The graph of $f(t)$ is given below:

+ Add content

Differential Equations Fall 2022

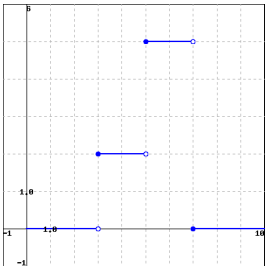
Home

Instructor Courses

Course Catalog

Problem Repository

Help Center



(Click on graph to enlarge)

a. Represent $f(t)$ using a combination of Heaviside step functions. Use $h(t-a)$ for the Heaviside function shifted a units horizontally.

$f(t) =$

b. Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ for $s \neq 0$.

=

☐ 23 (1 point) Given that

$$\mathcal{L}\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}$$

where $J_0(t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2}\left(\frac{t}{2}\right)^{2n}$, find the Laplace transform of $tJ_0(t)$.

$\mathcal{L}\{tJ_0(t)\} =$.

=

☐ 24 (1 point) Consider the following initial value problem:

$$y'' + 81y = \begin{cases} 5, & 0 \le t \le 6 \\ 0, & t > 6 \end{cases} \qquad y(0) = 2,\ y'(0) = 0$$

Using Y for the Laplace transform of $y(t)$, i.e., $Y = \mathcal{L}\{y(t)\}$, find the equation you get by taking the Laplace transform of the differential equation and solve for

$Y(s) =$

=

☐ 25 (1 point) Given that

$$\mathcal{L}\{\frac{\cos(9t)}{\sqrt{\pi t}}\} = \frac{e^{-9/s}}{\sqrt{s}}$$

find the Laplace transform of $\sqrt{\frac{t}{\pi}}\cos(9t)$.

$\mathcal{L}\{\sqrt{\frac{t}{\pi}}\cos(9t)\} =$.

=

26

Consider the function

$$f(t) = \begin{cases} 0 & \text{if } 0 \le t < 5 \\ 5 & \text{if } 5 \le t < 7 \\ 0 & \text{if } 7 \le t < \infty. \end{cases}$$

a. Use the graph of this function to write it in terms of the Heaviside function. Use $h(t-a)$ for the Heaviside function shifted a units horizontally.

$f(t) =$

b. Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ for $s \neq 0$.

$F(s) = \mathcal{L}\{f(t)\} =$

+ Add content

Differential Equations Fall 2022

Home

Instructor Courses

3

Course Catalog

Problem Repository

Help Center