1. Determine whether the following lines intersect, and if so, find the point of intersection: $\mathbf{r}_1(t) = \langle 1, -2, 4\rangle + t\langle 1, 3, -1\rangle$ and $\mathbf{r}_2(t) = \langle 0, 3, -3\rangle + t\langle 2, 1, 4\rangle$.

2. Let $\mathbf{v} = \langle -2, 3, 1\rangle$ and $\mathbf{w} = \langle 1, 0, -1\rangle$.

   (a) Find a unit vector orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

   (b) Find the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$.

3. Find the projection of $\mathbf{w} = \langle -3, 1, 0\rangle$ along $\mathbf{v} = \langle 1, 0, 1\rangle$, and then find the decomposition of $\mathbf{w}$ with respect to $\mathbf{v}$.

4. Find the two foci and the four vertices of the ellipse $x^2 + 25y^2 = 25$.

5. Find the position vector $\mathbf{r}(\mathrm{t})$ and the velocity vector $\mathbf{v}(\mathrm{t})$ if the acceleration vector is $\mathbf{a}(t) = \langle 4t, 6t\rangle$ and we know $\mathbf{v}(0) = \langle 1, -1\rangle$ and $\mathbf{r}(0) = \langle 1, 0\rangle$. Check your answer.

6. Describe in general terms the contour map for a nonconstant function of the form $f(x, y) = ax + by + c$ if a, b, and c are constants.

7. (a) Find a nonzero vector normal to the graph $z = 4 - x - 2y^2$ at the point $(x, y, z) = (0, 1, 2)$.

   (b) Use the linear approximation of $f(x, y) = 4 - x - 2y^2$ at $(0, 1)$ to estimate $f(0.01, 0.98)$.

8. Let $f(x, y, z) = 5x^2 - 3xy + xyz$.

   (a) Find the directional derivative of $f(x, y, z)$ in the direction of the vector $\langle 1, 1, -1\rangle$ at the point $P = (1, 3, 0)$.

   (b) In which direction (expressed as a unit vector) does $f(x, y, z)$ increase most rapidly at the point $P = (1, 3, 0)$?

   (c) What is this maximum rate of increase at $P = (1, 3, 0)$?

9. Find the critical points of the function $f(x, y) = x^4 + y^4 - 4xy + 1$. Then use the Second Derivative Test to determine whether they are local minima, local maxima, or saddle points (or state that the test fails.)

10. If D is the region bounded by the line $y = 2x$ and the parabola $y = x^2$, evaluate

$$\iint_D x\, dA.$$

11. Pick a suitable coordinate system for setting up the integral

$$\iiint_D \left(x^2 + y^2\right)^{3/2} \, dV,$$

where D is the region that lies above the cone $z\sqrt{3} = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 36$. Do not evaluate the integral.

12. Let $f(x, y, z) = e^x \cos(yz)$, and let $\mathbf{F} = \nabla f$. If $\mathbf{r}$ is any path from (0,0,0) to $(1, \pi^2, 1/\pi)$, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

13. Use Green's Theorem to evaluate $\oint_C xy^2 dx + x^3 dy$ if C is the boundary of the rectangle with vertices $(0, 0)$, $(2, 0)$, $(2, 3)$, and $(0, 3)$, oriented counterclockwise.

14. Use Green's Theorem to calculate the area enclosed by the circle $x^2 + y^2 = 16$.