# Final Exam Review Problems – Dr. Emory
# MATH 2153: Calculus II
# – Fall 2022

1. Write out the partial fraction decomposition of each function. Do not determine the numerical values of the coefficients.
   (a) $\dfrac{2}{(x-1)(x+1)} =$ ____________________
   (b) $\dfrac{4x^2+3x-1}{x(x^2+1)} =$ ____________________
   (c) $\dfrac{2x}{(x-1)(x-2)^2} =$ ____________________
2. Evaluate
$$\int \sin^2 x \cos^3 x \, dx$$
3. Use a trig substitution to evaluate
$$\int \frac{dt}{t^2\sqrt{t^2-16}}.$$
4. Evaluate the integral using partial fractions:
$$\int \frac{5x+1}{(2x+1)(x-1)} \, dx$$
5. Use the Comparison Theorem to determine whether the integral is convergent or divergent.
$$\int_1^\infty \frac{3+\sin x}{\sqrt{x}} \, dx$$
6. Determine if the integral is convergent or divergent. Evaluate if convergent.
$$\int_3^\infty \frac{dx}{(x-2)^{3/2}}$$
7. Evaluate
$$\int \frac{dx}{(1-x^2)^{3/2}}$$
8. Evaluate
$$\int_1^3 r^4 \ln r \, dr$$

9. Evaluate

$$\int te^{-3t}\ dt$$

10. Use integration by parts to evaluate

$$\int e^{-\theta}\cos 2\theta\ d\theta.$$

11. (12 points) Determine if the series converges or diverges using the $n$th Divergence Test.

   (a) (8 points) Calculate the limit.

$$\lim_{n\to\infty}\frac{n}{\sqrt{n^2+16}}$$

   (b) (4 points) Use this limit to determine if the series $\sum_{n=1}^{\infty}$ converges or diverges.

   Does the series converge or diverge? **Circle the correct answer**.

   1. Because the limit is finite, the series converges by the $n$th Term Divergence Test.
   2. Because the limit is infinite, the series diverges by the $n$th Term Divergence Test.
   3. Because the limit is finite and nonzero, the series diverges by the $n$th Term Divergence Test.

12. Calculate the arc length, $s$ of the function $y = 12x^{3/2}$ over the interval $[1, 2]$.

13. A plate in the shape of an isosceles triangle with base 1 meter and height 10 meters is submerged vertically in a tank of water so that the top of the triangle is located 3 m below the surface of the water. Calculate the total fluid force $F$ on a side of the plate. The acceleration for gravity is 9.8 $m/s^2$ and the density of water is 1000 $kg/m^3$.

14. Compute the surface area of revolution about the $x$-axis over the interval $[0, 8]$ for $y = x$.

15. Consider the series.

$$\frac{14}{3}+\frac{14}{3^2}+\frac{14}{3^3}+\frac{14}{3^4}+\cdots$$

   This can be written as a geometric series in the form $\sum_{n=0}^{\infty} cr^n$. Identify $c$ and $r$ in the geometric series.

   $c =$ ____________, $r =$ ____________

Calculate the sum of the series

$$\frac{14}{3}+\frac{14}{3^2}+\frac{14}{3^3}+\frac{14}{3^4}+\cdots=$$

16. Calculate $S_3$, $S_4$, and $S_5$, and then find the sum $\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$ using the identity

$$\frac{1}{4n^2-1}=\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$$

$S_3$=______________________________

$S_4$=________________________________________________

$S_5$=__________________________________________________________

$\sum_{n=1}^{\infty}\frac{1}{4n^2-1}=$ ____________________

17. Determine if the series converges or diverges. Find the sum if possible.

$$\sum_{n=2}^{\infty}e^{1-4n}$$

18. Determine the limit of the sequence and state if the sequence converges or diverges

$$a_n=\ln\left(\frac{2n+9}{-8+5n}\right)$$

19. Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty}\frac{n}{10n+12}$$

20. Use the Squeeze Theorem to determine the limit of the sequence

$$a_n=\frac{\sin n}{\sqrt{n}}$$

21. Use the Limit Comparison Test to test the series for convergence or divergence

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2-1}}$$

22. Use the Integral Test to determine if the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

23. Find the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$$

24. Use the Root Test to test the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+3}{3n+2}\right)^n$.

25. Compute the 3rd degree Taylor polynomial, $T_3$ for $f(x) = 3\sqrt{x}$ centered at $a = 1$.

26. Determine whether the series is absolutely convergent, conditionally convergent, or divergent $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+4}$.

27. Evaluate the indefinite integral as an infinite series

$$\int \frac{\cos x - 1}{x}\, dx$$

28. Use series to evaluate the limit. (Do not use l'Hopitals rule)

$$\lim_{x\to 0} \frac{x - \ln(1+x)}{x^2}$$

29. Test the series for convergence or divergence

$$\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

30. Test the series for convergence or divergence

$$\sum_{n=1}^{\infty} \frac{1}{5+4^n}$$

31. Find $\dfrac{dy}{dx}$ for $\left(\ln(t), \dfrac{1}{t}\right)$ at $t = 7$.

32. Find $\dfrac{dy}{dx}$ for $(\sec\theta, \tan\theta)$ at $\theta = \dfrac{3\pi}{4}$.

33. Calculate the arc length integral $s$ for the logarithmic spiral

$$c(t) = (e^t \cos(t), e^t \sin(t))$$

for $0 \le t \le 7$

34. Compute the surface area of the cone generated by revolving $c(t) = (t^2, t)$ for $0 \le t \le 2$

35. Convert to an equation in rectangular coordinates

$$r = 3\csc(\theta) - \sec(\theta)$$

36. Convert the equation $r = 4\sec(\theta)$ from polar coordinates to rectangular coordinates