Math 2144 Exam 1 **Spring 2024** **NAME:**

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an approved calculator to

- perform operations on real numbers,
- evaluate functions at specific values, and
- generate and examine at graphs and/or tables.

A TI-89, TI-Nspire, or any calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Having your phone out for any reason during the exam is an academic integrity violation. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and, when prompted, explain your reasoning to earn full credit. For those tasks that explicitly prompt you to show work or explain your reasoning, answers alone will receive no credit. The purpose of this assessment is for you to demonstrate what you know.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 12 pages. Make sure all problems and pages are present.

Please turn in your notecard with the exam. Make sure your name is on your notecard.

The exam is worth 57 points in total.

You have **60 minutes** to work starting from the signal to begin.

This page is intentionally blank. You may use it for scratch work, but any work recorded on this page will not be graded.

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) The expression $\lim_{h\to 0} \frac{(x+h)^3 - \ln(x+h) - (x^3 - \ln(x))}{h}$ is the derivative of what function?

   a. $f(x) = (x+h)^3 - \ln(x+h)$

   b. $f(x) = 3x^2 - \frac{1}{x}$

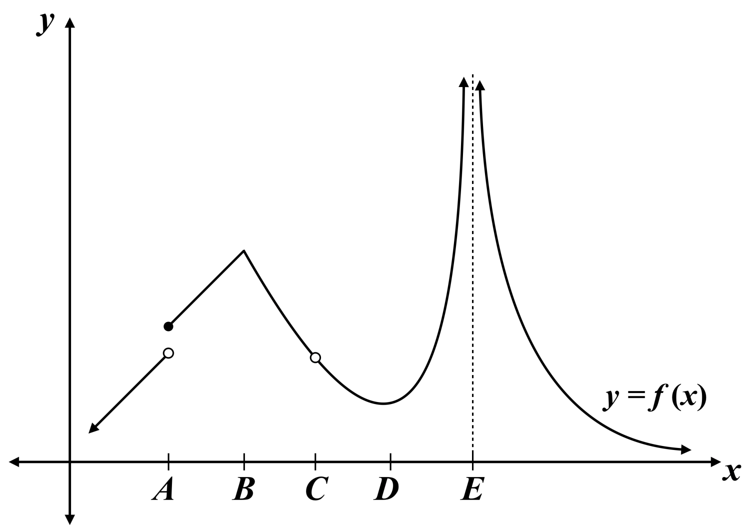
   c. $f(x) = (x+h)^3 - \ln(x+h) - (x^3 - \ln(x))$

   d. $f(x) = x^3 - \ln(x)$

   e. $f(x) = \frac{(x+h)^3 - \ln(x+h) - (x^3 - \ln(x))}{h}$

   (ii) The graph of the function $y = f(x)$ is below. For which value of K do the one-sided limits $\lim_{x\to K^+} \frac{df}{dx}$ and $\lim_{x\to K^-} \frac{df}{dx}$ exist but are not equal?

   (*Only one answer is correct.*)



   a. A

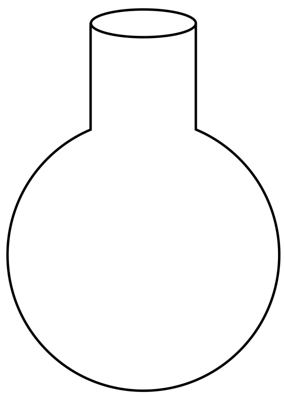
   b. B

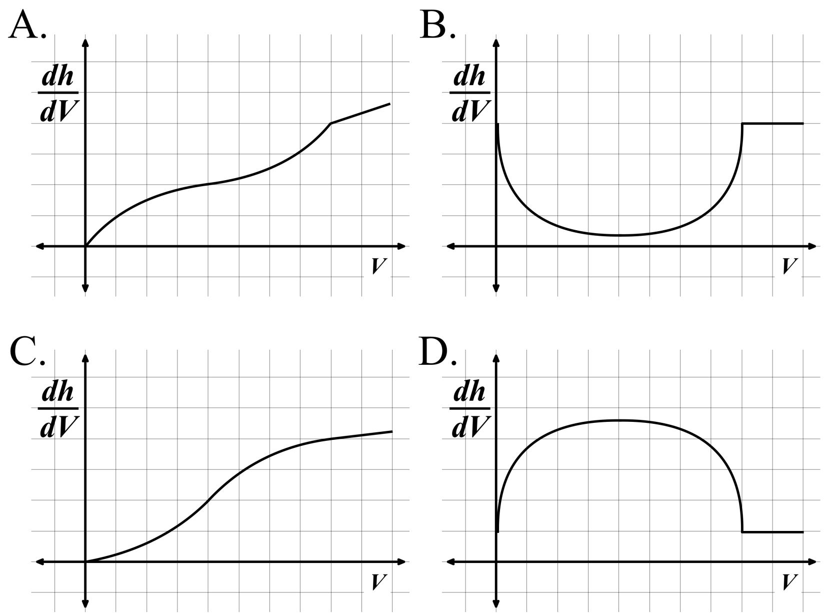
   c. C

   d. D

   e. E

(iii) Imagine the vase pictured below filling with water. Let V represent the volume of water in the vase and let h represent the height of water in the vase. Which of the following graphs could represent the function $y = \dfrac{dh}{dV}$, the **derivative** of the function h with respect to V (not the function $y = h(V)$)?





a. Graph A

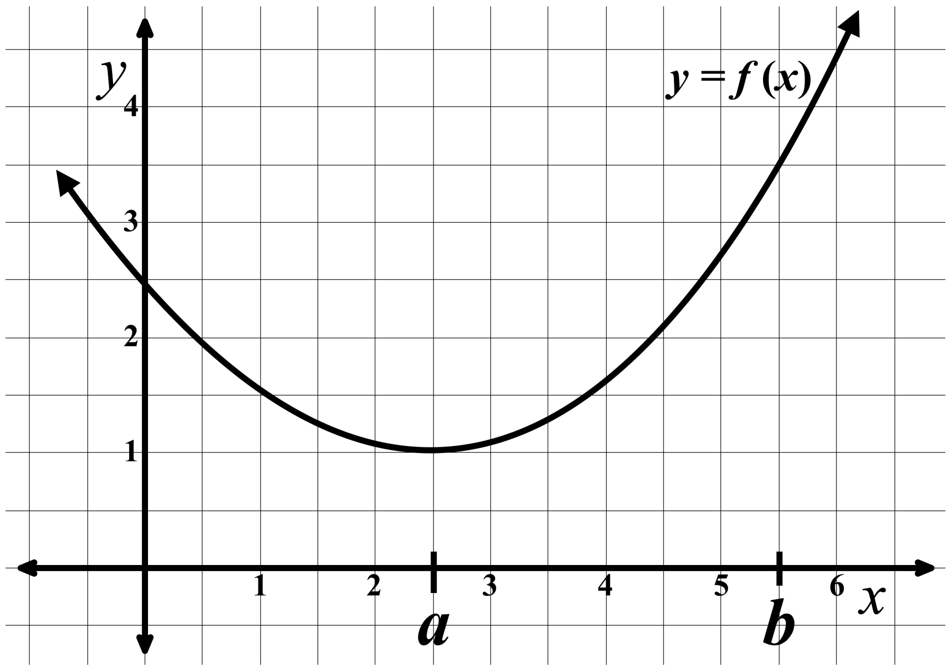
b. Graph B

c. Graph C

d. Graph D

e. None of these

(iv) The graph of the function $y = f(x)$ is below. Note that the scales on the x and y axes are the same. Which of the following inequalities is true?



a. $\dfrac{f(b)-f(a)}{b-a} < f(b)-f(a) < \lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

b. $\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h} < f(b)-f(a) < \dfrac{f(b)-f(a)}{b-a}$

c. $\dfrac{f(b)-f(a)}{b-a} < \lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h} < f(b)-f(a)$

d. $\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h} < \dfrac{f(b)-f(a)}{b-a} < f(b)-f(a)$

e. $f(b)-f(a) < \dfrac{f(b)-f(a)}{b-a} < \lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

(v) Which of the following expressions represents the derivative of the function $y = f(x)$?

I. $\dfrac{f(x) - f(a)}{x - a}$

II. $\dfrac{f(x + 0.00001) - f(x)}{0.00001}$

III. $\lim\limits_{h \to 0} \dfrac{f(x) - f(a)}{x - a}$

IV. $\lim\limits_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

V. $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

a. I and V only

b. II only

c. III and IV only

d. IV only

e. IV and V only

2. (*8 points*) Consider the function f defined by

$$f(x) = \begin{cases} 6 - x^2 & x \leq 2 \\ mx + b, & x > 2 \end{cases}$$

where m and b are real numbers.

(a) (*2 points*) Compute $\frac{d}{dx}\left(6 - x^2\right)$. (*You may use the power rule to differentiate this expression*: $\frac{d}{dx}x^n = nx^{n-1}$.)

(b) (*2 points*) Compute $\frac{d}{dx}(mx + b)$ in terms of the constants m and b.

(c) (*4 points*) Determine values of constants m and b that make f continuous ***and*** differentiable at $x = 2$. **Justify your response** by demonstrating that the conditions of continuity and differentiability are satisfied.

3. (*3 points each*) In electrical engineering, the Shockley diode equation expresses the relationship between the current (I) flowing through a diode and the voltage (V) across it. This equation is defined as

$$I = f(V) = I_s\left(e^{\frac{V}{nV_T}} - 1\right)$$

where I represents the diode current (in microamps), I_s represents the reverse saturation current (a constant for a given diode at a fixed temperature), V is the voltage across the diode (in volts), n is the ideality factor (a constant), and V_T is the thermal voltage (a constant).

Describe the meaning of the following expressions. Your response should identify the *quantity* represented by the expression. **Specify the units** associated with each expression.

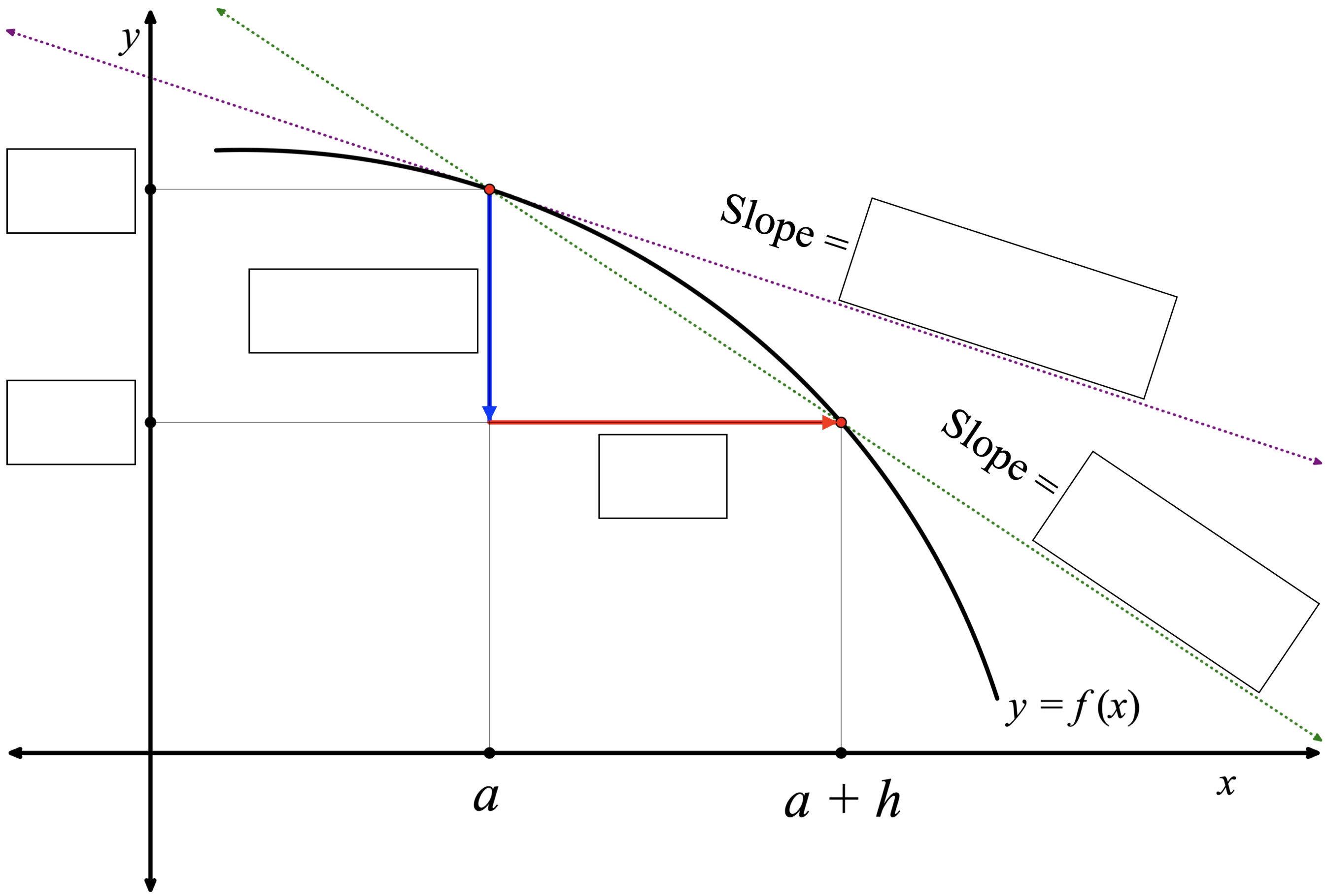
(a) $f(0.71) - f(0.65)$.

(b) $\dfrac{f(0.71) - f(0.65)}{0.71 - 0.65}$.

(c) $\displaystyle\lim_{\Delta V \to 0} \frac{f(0.65 + \Delta V) - f(0.65)}{\Delta V}$

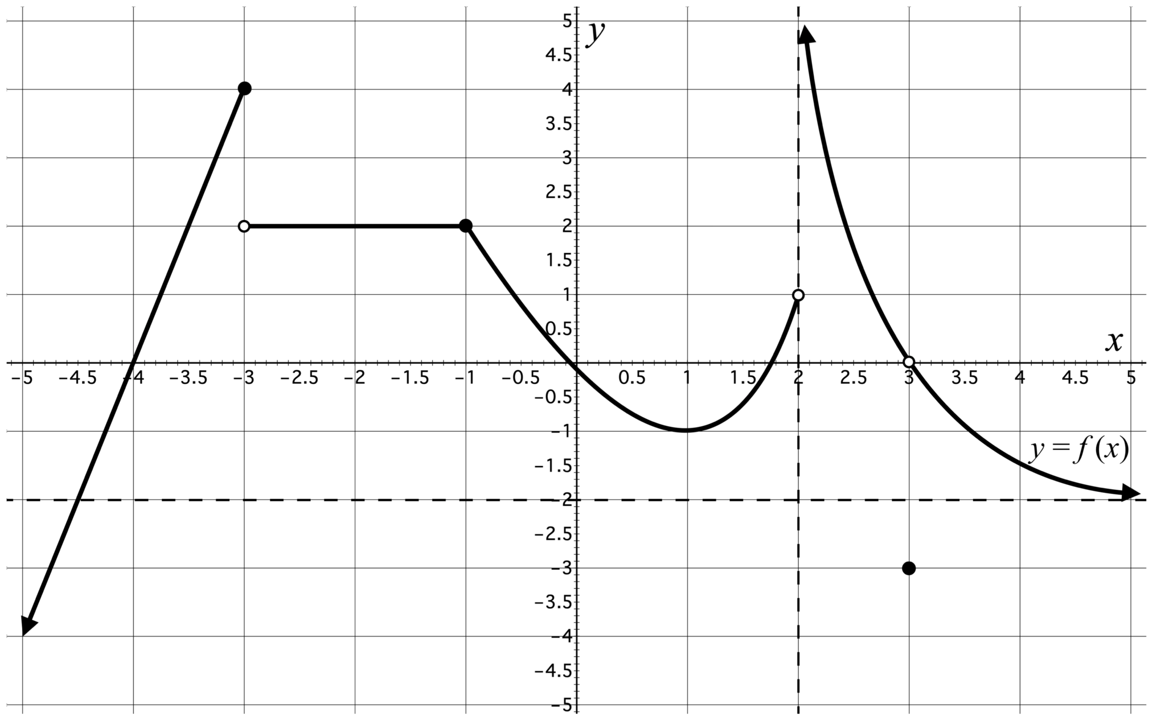
(d) What does the solution to the equation $f'(V) = 0.013$ represent?

4. (*6 points*) Consider the graph of $y = f(x)$ illustrated below.



Write an expression in each of the six boxes that represents the respective graphical quantity.

5. (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all maxima and minima, points of discontinuity, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "∞" or "$-\infty$" as appropriate. Write "DNE" if the value does not exist.

$\lim_{x\to -3^+} f(x) =$

$f'(-4) =$

$\lim_{x\to 2^+} f(x) =$

$\lim_{\Delta x\to 0} \frac{f(-3.7+\Delta x)-f(-3.7)}{\Delta x} =$

$\lim_{x\to 3} f(x) =$

$\left.\frac{df}{dx}\right|_{x=-1} =$

$\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} =$

$\lim_{x\to -3^-} \frac{df}{dx} =$

$\lim_{x\to -1} f(x) =$

6. (*3 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. *Note that in Part (b), k is a constant.*

   (a) $\displaystyle\lim_{x\to 1} \frac{x^2e^x - ex^2}{e^x - e} =$

   6(a) answer:

   (b) $\displaystyle\lim_{x\to k^-} \frac{|x-k|}{x-k} =$

   6(b) answer:

7. (*6 points*) Use the limit definition of derivative to show that

$$\frac{d}{dx}\left(3x^2 - 5x\right) = 6x - 5.$$

You **must** use the **limit definition of derivative** to receive credit.