Math 2144 Exam 2 **Spring 2024** **NAME:** ______________________

CRN: ______________________

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an approved calculator to

- perform operations on real numbers,
- evaluate functions at specific values, and
- generate and examine at graphs and/or tables.

A TI-89, TI-Nspire, or any calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Having your phone out for any reason during the exam is an academic integrity violation. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and, when prompted, explain your reasoning to earn full credit. For those tasks that explicitly prompt you to show work or explain your reasoning, answers alone will receive no credit. The purpose of this assessment is for you to demonstrate what you know.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 11 pages. Make sure all problems and pages are present.

Please turn in your notecard with the exam. Make sure your name is on your notecard.

The exam is worth 72 points in total.

You have **60 minutes** to work starting from the signal to begin.

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1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Assume the following information about the function f:

   - $f(6) = 11$
   - $\left.\dfrac{df}{dx}\right|_{x=6} = f'(6) = \dfrac{1}{4}$
   - $\dfrac{d^2 f}{dx^2} > 0$ for $6 \leq x \leq 6.5$

   Based on only this information, what is the best linear approximation of $f(6.5)$, and is the approximation an overestimate or underestimate?

   a. 11.25 is an underestimate of $f(6.5)$

   b. 11.25 is an overestimate of $f(6.5)$

   c. 2.75 is an overestimate of $f(6.5)$

   d. 2.75 is an underestimate of $f(6.5)$

   e. 11.125 is an underestimate of $f(6.5)$

   f. 11.125 is an overestimate of $f(6.5)$

   (ii) A 3,000 in^3 volume box container with a square base is being constructed. The material to construct the top and bottom (square) faces cost 3 cents per square inch, and the side faces each have a cost of cost 2 cents per square inch. Let x be the variable lengths for the base sides and h be the variable height length so that the box has dimensions $x \times x \times h$. By solving which of the following equations can we determine the value of x for which the cost (C) of the box is minimized?

   a. $C(x) = 6x^2 + \dfrac{24000}{x}$

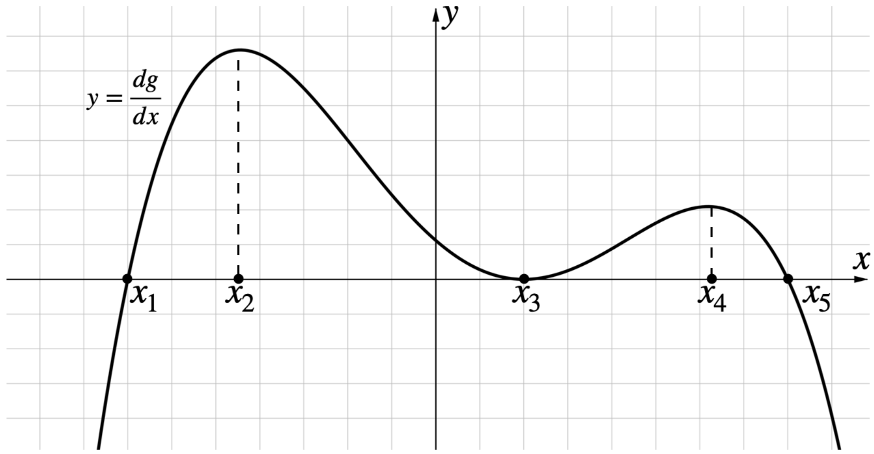
   b. $0 = 6x^2 + \dfrac{24000}{x}$

   c. $C'(x) = 12x - \dfrac{24000}{x^2}$

   d. $0 = 12x - \dfrac{24000}{x^2}$

   e. $C''(x) = 3000$

(iii) The graph of the **derivative** of g, $y = g'(x)$, is shown below. Identify which of the following statements are true.



I. x_2 and x_4 are critical points of g.

II. $g(x_3)$ is a local extreme value (either maximum or minimum) of g.

III. x_1, x_3, and x_5 are critical points of g.

IV. $(x_2,\ g(x_2))$, $(x_3,\ g(x_3))$, and $(x_4,\ g(x_4))$ are inflection points of g.

V. $g(x_1)$ is a local minimum of g.

a. I only

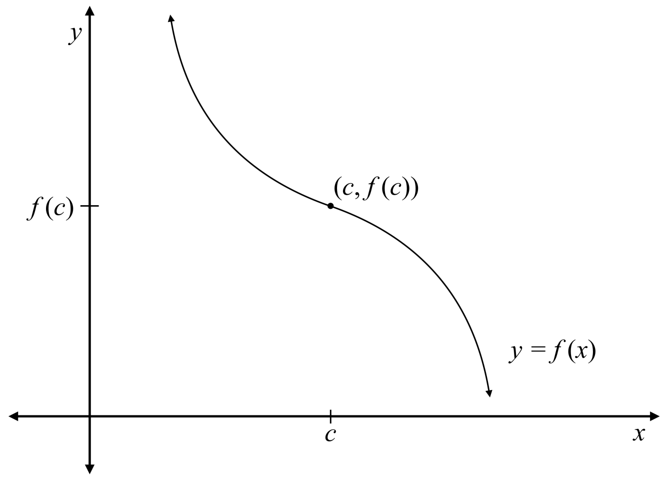
b. I and II only

c. III and V only

d. II and III only

e. III, IV, and V only

(iv) A portion of the graph of a function f that passes through the point $(c,\ f(c))$ is shown below. Which of the following conditions are satisfied?



a. $f'(c) > 0$, $f''(x) > 0$ for all $x < c$, and $f''(x) < 0$ for all $x > c$

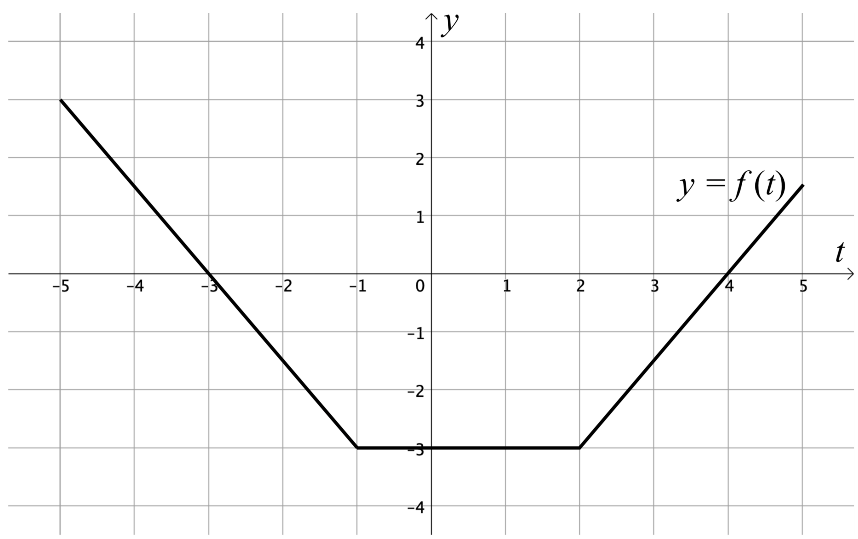
b. $f'(c) > 0$, $f''(x) < 0$ for all $x < c$, and $f''(x) > 0$ for all $x > c$.

c. $f'(c) < 0$, $f''(x) > 0$ for all $x < c$, and $f''(x) < 0$ for all $x > c$.

d. $f'(c) < 0$, $f''(x) < 0$ for all $x < c$, and $f''(x) > 0$ for all $x > c$.

e. None of the above.

(v) The graph of $y = f(t)$ is shown below.



Given $g(t) = \dfrac{t^2}{f(t)}$, what is $g'(1)$?

a. $\dfrac{-2}{3}$ b. $\dfrac{2}{9}$ c. $\dfrac{-1}{2}$ d. $\dfrac{-7}{9}$

e. $g'(1)$ is undefined

2. (*8 points*) Use calculus to determine the following for the function

$$f(x) = 2x^3 + 3x^2 - 12x + 4.$$

   (a) (*6 points*) Find the critical points of f. Work must be shown to receive credit.

   (b) (*2 points*) Determine the absolute maximum and absolute minimum of f on the interval $[0, 2]$. Work must be shown to receive credit.

3. (*5 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L'Hôpital's Rule, clearly indicate when you do so**.

   (a) $\displaystyle \lim_{x \to 0} \left(\frac{\sin(x)}{e^x - 1} \right) =$

   (b) $\displaystyle \lim_{x \to \infty} \left(\frac{x^2 - 9x^3}{e^x + \sin(x)} \right) =$

   (c) $\displaystyle \lim_{x \to (\pi/2)^-} \left(\frac{\sec(x)}{1 + \tan(x)} \right) =$

4. (*5 points each*) Use the table of values below to answer the following questions. State your answers as exact values.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	7	-1	3	6
2	5	1	3	-4
4	0	1	0	-2

(a) If $h(x) = f((g(x))^2)$, then $h'(2) =$

(b) If $s(x) = x^2 \arctan(f(x))$, then $s'(4) =$

(c) If $k(x) = 2^{g(x)}$, then $k'(1) =$

5. (*4 points each*) Compute the following derivatives.

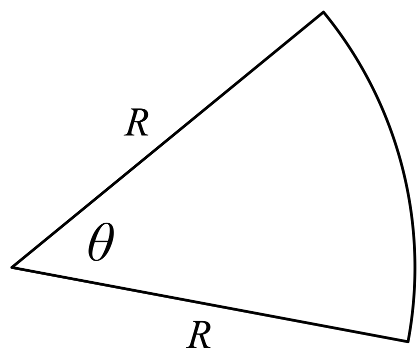
   (a) Let $f(x) = \pi x^{e^{\pi}}$. Find $f'(x)$.

   (b) Let $y = \sin(\theta)$ where θ is measured in degrees (not radians!). Find the **second derivative** $\left(\frac{d^2y}{d\theta^2}\right)$, or y'', of y with respect to θ.

6. (*6 points*) Find $\dfrac{dy}{dx}$ for the curve $y - \sec(4^y) = xy + \sin(x)$. Solve for $\dfrac{dy}{dx}$ in terms of both x and y.

7. (*10 points*) Nathaniel wants to construct a garden in the shape of a sector that will be bordered by brick pavers (see the image below). The area of the garden will be 60 square feet. R represents the radius of the sector and θ represents the measure of the interior angle of the garden (in radians), as shown in the image below. Determine the values of R and θ that will minimize the amount of pavers Nathaniel needs to buy to create his garden. Justify that these dimensions minimize the perimeter of the garden. **Express your solution as exact values, not decimals.**

   Recall that the area A of a circle of radius r is given by $A = \pi r^2$. Also recall that an angle's measure in radians represents the length of a subtended arc measured in units of the arc's radius.



DERIVATIVE FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$