Math 2144 Exam 2 **Fall 2022** **NAME:** ____________________

CRN: ____________________

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5 inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or any calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Having your phone out for any reason during the exam is an academic integrity violation. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. Reasoning which will earn credit will use material covered in the course to date.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 9 pages. Make sure all problems and pages are present.

Please turn in your notecard with the exam. Make sure your name is on your notecard.

The exam is worth 58 points in total.

You have **60 minutes** to work starting from the signal to begin.

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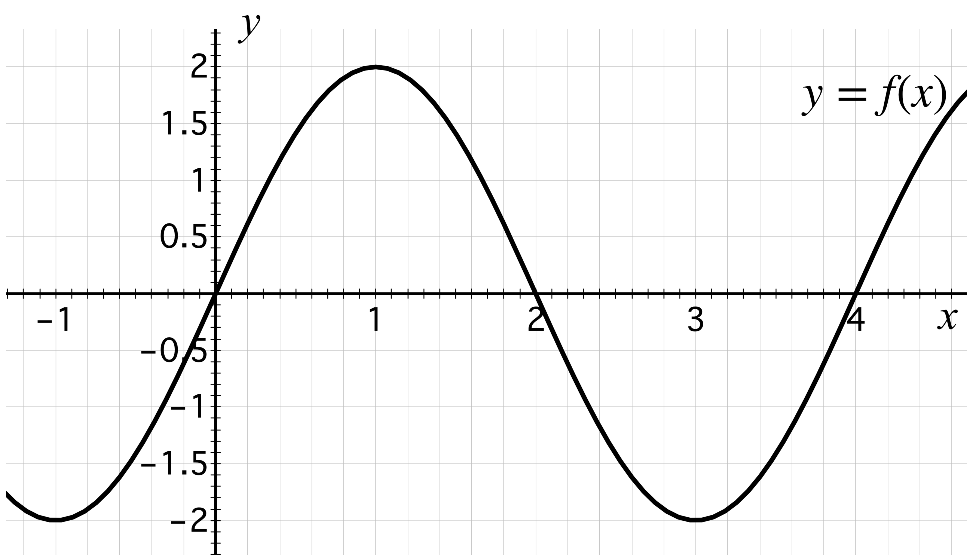
1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) The graph of the function $y = f(x)$ is displayed below. For which value(s) of c is the following true? Select all that apply.

$$\left.\frac{df}{dx}\right|_{x=c} < \left.\frac{d^2 f}{dx^2}\right|_{x=c}$$



   a. $c = 0$

   b. $c = 1$

   c. $c = 2$

   d. $c = 3$

   e. $c = 4$

   (ii) Suppose that f is continuous and differentiable on the interval $[3,\ 10]$. Also suppose that $f(3) = 4$ and $\frac{df}{dx} = f'(x) \geq 2$ for all x in the interval $[3,\ 10]$. What is the smallest possible value for $f(10)$?

   a. 18

   b. 14

   c. 8

   d. 11

   e. 17

(iii) Assume the following information about the function f:

- $f(6) = 11$
- $\left.\dfrac{df}{dx}\right|_{x=6} = f'(6) = \dfrac{1}{4}$
- $\dfrac{d^2 f}{dx^2} > 0$ for $6 < x < 6.5$

Based on only this information, what is the best linear approximation of $f(6.5)$ and is the approximation an overestimate or underestimate?

a. 11.25 is an underestimate of $f(6.5)$

b. 11.25 is an overestimate of $f(6.5)$

c. 2.75 is an overestimate of $f(6.5)$

d. 2.75 is an underestimate of $f(6.5)$

e. 11.125 is an underestimate of $f(6.5)$

f. 11.125 is an overestimate of $f(6.5)$

(iv) Suppose $g'(3) = 0$, $g''(x) = x(1-x)^2$, and $g(x)$ is defined for all $x \in (-\infty, \infty)$. Which of the following is true?

I. $g(3)$ is a local maximum

II. $g(3)$ is a local minimum

III. g is concave down on the interval $(-\infty,\ 0)$

IV. g changes concavity at $x = 0$ and $x = 1$

a. I only

b. II only

c. I and IV only

d. II and III only

e. II and IV only

2. (*6 points*) Find the critical points of $f(x) = 2x^4 - 5x^3 + 3$ and classify them as corresponding to local maxima, local minima, or neither. Justify your classification of the critical points without referring to a calculator-generated graph.

3. A differentiable function $y = g(x)$ is defined with the following properties:

   (i) g is increasing for all x;

   (ii) g is concave down for all x;

   (iii) $g(6) = 4$;

   (iv) $\left.\dfrac{dg}{dx}\right|_{x=6} = g'(6) = 2$.

   (a) (*3 points*) Explain why g has exactly one zero (i.e., explain why there is only one value of x for which $g(x) = 0$.)

   (b) (*3 points*) Explain why the zero (i.e., $x-$intercept) of g **must** be in the interval (4, 6).

4. (*4 points each*) Compute the following derivatives. Do not simplify.

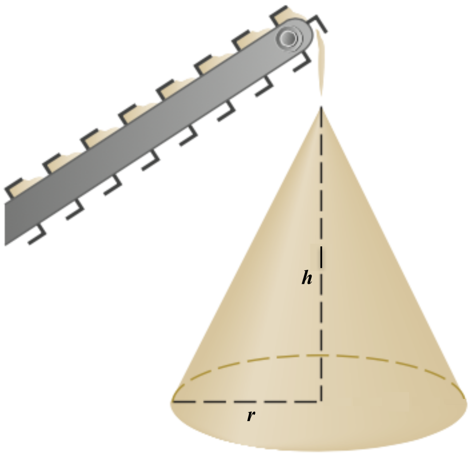
   (a) Let $f(t) = \sec(\sqrt{2t})$. Find $f'(t)$.

   (b) Let $y = 5^{\arcsin(x)}$. Find $\frac{dy}{dx}$. (*Recall that* $\arcsin(x) = \sin^{-1}(x)$, *the inverse of the sine function).*

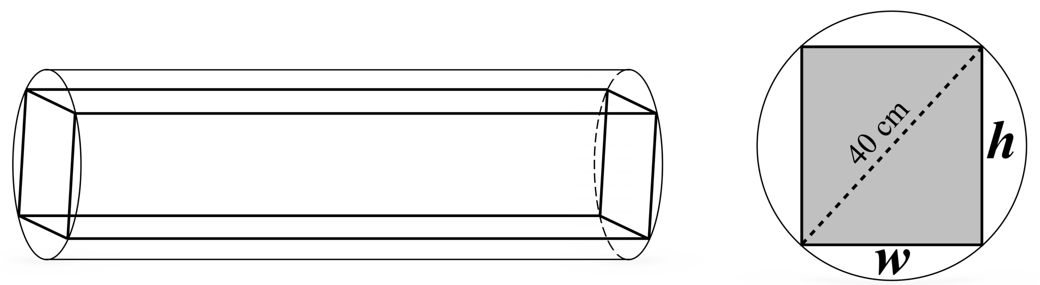
   (c) Let $g(t) = x\ln(x)$. Find the **second derivative** of g $\left(\frac{d^2g}{dt} \text{ or } g''(x)\right)$.

5. (*6 points*) Find $\frac{dy}{dx}$ for the curve $xe^y = \tan(y)$. Your answer for $\frac{dy}{dx}$ should contain both x and y.

6. (*10 points*) Suppose gravel is being poured into a conical pile at a rate of 5 m^3/s, and suppose that the radius r of this cone is always half its height h. How fast is the height of the pile increasing when the height is 10 m?
(*Note that the formula for the volume of a cone is* $V = \frac{1}{3}\pi r^2 h$*).*



7. (*10 points*) Suppose a rectangular beam is cut from a cylindrical log of diameter 40 cm as shown in the image below. The strength of a beam, S, is given by the formula $S = 7wh^2$, where w represents the width of the beam in centimeters and h represents the height of the beam in centimeters. Find the width and height of the beam with maximum strength that can be cut from the log. Justify your answer.



Cross-section

BASIC FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$