Math 2144 Final **Spring 2023** **NAME:** ______________________

CRN: ______________________

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an approved calculator to perform operations on real numbers, evaluate functions at specific values, and look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e., laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work**, including all steps needed to solve each problem and explain your reasoning to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. This comprehensive exam assesses your understanding of material covered throughout the entire course.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 10 problems on 13 pages (including the page containing basic formulas). Make sure all problems and pages are present.

The exam is worth 85 points in total.

You have **1 hour** and **50 minutes** to work starting from the signal to begin.

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1. (*2 points each*) Answer the following multiple choice questions. No justification or explanation is required. ***Record your answers in the spaces provided on page4 of the exam.***

   (i) The table below gives selected values for the differentiable function f. In which of the following intervals must there be a value c such that $f'(c) = \frac{1}{2}$?

x	0	2	3	5	8	10	11	15
$f(x)$	-3	-6	0	4	1	2	8	6

   A. $[0,\ 2]$

   B. $[3,\ 5]$

   C. $[5,\ 8]$

   D. $[8,\ 10]$

   E. $[11,\ 15]$

   (ii) The expression $\displaystyle\sum_{k=1}^{30}\left(5+\frac{k}{5}\right)^7\cdot\frac{1}{5}$ is a Riemann sum approximation of which of the following integrals?

   A. $\displaystyle\frac{1}{5}\int_1^{30}\left(5+\frac{x}{5}\right)^7\,dx$

   B. $\displaystyle\int_1^{30}\frac{x^7}{5}\,dx$

   C. $\displaystyle\frac{1}{5}\int_1^{30}(5+x)^7\,dx$

   D. $\displaystyle\int_5^{11}x^7\,dx$

   E. $\displaystyle\frac{1}{5}\int_5^{11}x^7\,dx$

   (iii) Evaluate $\displaystyle\lim_{h\to 0}\frac{\cos\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}\right)}{h}$.

   A. 0

   B. 1

   C. -1

   D. π

   E. $-\pi$

(iv) Suppose $\int_2^4 f(x)\,dx = 10$ and $\int_2^4 g(x)\,dx = -5$. Which of the following expressions **cannot** be determined from this information alone?

A. $\int_4^2 f(x)\,dx$

B. $\int_2^4 \frac{g(x)}{f(x)}\,dx$

C. $\int_2^4 7g(x)\,dx$

D. $\int_2^4 \left(f(x) + 5g(x)\right)\,dx$

E. $\int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx$

(v) Consider the function f defined by

$$f(x) = \begin{cases} 3x^2 - 4, & x < 1 \\ 2, & x = 1 \\ 6x - 4, & x > 1 \end{cases}$$

Which of the following are true statements about this function?

I. $\lim_{x \to 1} f(x)$ exists

II. $f(1)$ exists

III. f is continuous at $x = 1$

A. I only

B. I and II

C. II only

D. II and III

E. I, II, and III

Record your answers to the five multiple choice questions in the spaces provided below. Write legibly and ***use capital letters.***

(i) ☐ (ii) ☐ (iii) ☐ (iv) ☐ (v) ☐

2. (*3 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numerical answers without justification will earn no credit. **If you use L'Hôpital's Rule, clearly justify why you are able to do so**. *Write your final answer in the space provided.*

   (a) $\displaystyle\lim_{x\to 8} \frac{x^3 - 64x}{x - 8} =$

   2(a) answer: $\boxed{}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\sin(x)}{e^x} =$

   2(b) answer: $\boxed{}$

3. (*4 points each*) Compute the following derivatives. **You do not need to simplify**.

   (a) Let $g(x) = 4^x + x^4 + \dfrac{4}{x}$. Find $g'(x)$.

   (b) Let $f(x) = \arctan(x)$. Find the **second derivative** of f, $\dfrac{d^2 f}{dx^2}$.

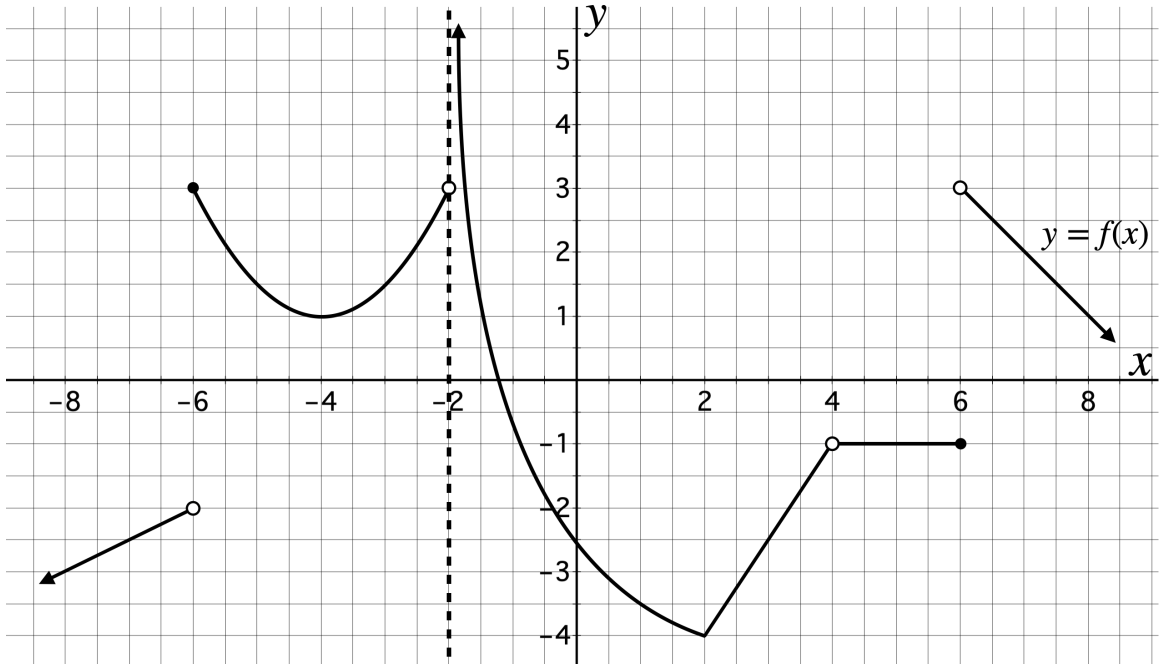
4. (*5 points*) Find $\dfrac{dy}{dx}$ for the curve $xe^{2y}+y = \ln(3)$ and evaluate $\dfrac{dy}{dx}$ at the point $(0, \ln(3))$. *Write your numerical answer in the space provided.*

Answer: $\boxed{}$

5. (*7 points*) Suppose a 5-foot-tall person is walking away from a 20-foot lamppost at a speed of 3 feet per second. At what rate is the **length of the person's shadow changing**? *Write your numerical answer in the space provided.*

Answer: $\boxed{}$ ft/sec

6(a) (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all critical points, points of discontinuity, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

$\lim_{x \to 4} f(x) =$

$\left.\dfrac{df}{dx}\right|_{x=-7} =$

$\int_5^6 f(x)\,dx =$

$f'(-4) =$

$\lim_{x \to -2^-} f(x) =$

$\lim_{\Delta x \to 0} \frac{f(8+\Delta x)-f(8)}{\Delta x} =$

$\lim_{x \to 2^+} \dfrac{df}{dx} =$

$\lim_{x \to 6} f(x) =$

$f''(7) =$

6(b) (*2 points*) Identify all values of x in the interval $(-8, 8)$ where f is continuous but not differentiable.

$x = \boxed{}$

7. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

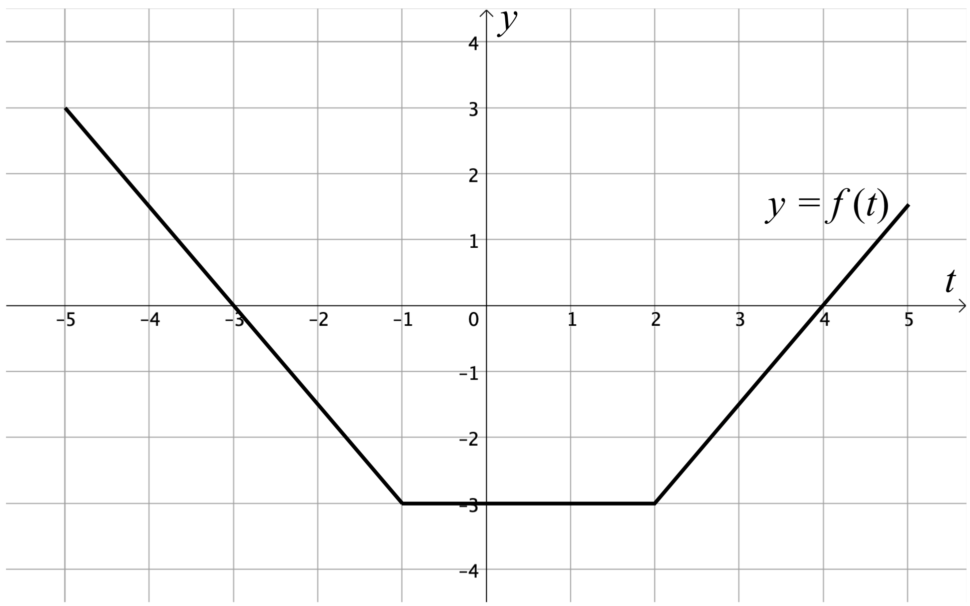
   (a) $\displaystyle\int \left(x^5 - \sin(x) + 5^x\right)\, dx =$

   (b) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos(\theta)}{1+\sin^2(\theta)}\, d\theta =$

   (c) $\displaystyle\frac{d}{dx}\int_1^{\sqrt{x}} \ln(4t)\, dt =$

8. (*8 points*) The graph of the function $y = f(t)$ is given below. *Note that the graph of f is defined for all x in the interval* $[-5,\ 5]$. Define $A(x)$ by

$$A(x) = \int_0^x f(t)\,dt.$$



(a) (*4 points*) Evaluate $A(1)$ and $A(-3)$.

$A(1) =$ $\boxed{}$

$A(-3) =$ $\boxed{}$

(b) (*2 points*) For what value(s) of x on the interval $[-5,\ 5]$ does $A(x)$ have a local maximum?

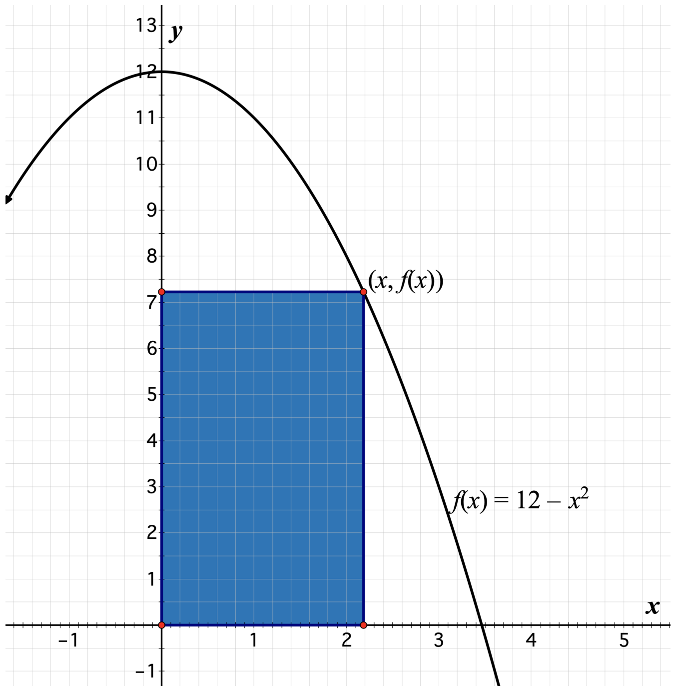
$x =$ $\boxed{}$

(c) (*2 points*) Find $A'(1)$ and $A''(4)$.

$A'(1) =$ $\boxed{}$

$A''(4) =$ $\boxed{}$

9. (*10 points*) What is the area of the largest rectangle in the first quadrant with one vertex on the origin and the opposite vertex on the graph of the function $f(x) = 12 - x^2$, as shown in the diagram below? *Write your numerical answer in the space provided.*



Answer: $\boxed{}$ square units

10. (*8 points*) The function A is given by $A(x) = \displaystyle\int_0^x \left(t^2 - 2t\right)\, dt$. Determine the coordinates of the point of inflection of the graph of $y = A(x)$.

Write the x and y coordinates in the spaces provided.

x-coordinate: $\boxed{}$ y-coordinate: $\boxed{}$

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BASIC FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$
$$\frac{d}{dx}e^x = e^x$$
$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$
$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$
$$\frac{d}{dx}a^x = a^x\ln(a)$$
$$\frac{d}{dx}\sin(x) = \cos(x)$$
$$\frac{d}{dx}\cos(x) = -\sin(x)$$
$$\frac{d}{dx}\tan(x) = \sec^2(x)$$
$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$
$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$
$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$
$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$
$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$
$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$
$$\int a^u\,du = \frac{a^u}{\ln a} + C$$
$$\int \sin(u)\,du = -\cos(u) + C$$
$$\int \cos(u)\,du = \sin(u) + C$$
$$\int \sec^2(u)\,du = \tan(u) + C$$
$$\int \csc^2(u)\,du = -\cot(u) + C$$
$$\int \sec(u)\tan(u)\,du = \sec(u) + C$$
$$\int \csc(u)\cot(u)\,du = -\csc(u) + C$$
$$\int \tan(u)\,du = \ln|\sec u| + C$$
$$\int \cot(u)\,du = \ln|\sin u| + C$$
$$\int \sec(u)\,du = \ln|\sec u + \tan u| + C$$
$$\int \csc(u)\,du = \ln|\csc u + \cot u| + C$$
$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$
$$\int f(u(x))\cdot u'(x)\,dx = \int f(u)\,du$$
$$L_n = \Delta x\sum_{k=1}^{n} f(a+(k-1)\Delta x)$$
$$R_n = \Delta x\sum_{k=1}^{n} f(a+k\Delta x)$$
$$M_n = \Delta x\sum_{k=1}^{n} f\left(a+\frac{2k-1}{2}\Delta x\right)$$