INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. This comprehensive exam assesses your understanding of material covered throughout the entire course.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 9 problems on 15 pages (including the page containing basic formulas). Make sure all problems and pages are present.

The exam is worth 88 points in total.

You have **1 hour** and **50 minutes** to work starting from the signal to begin. Good luck!

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Final Exam Grade by Problem Number

No.	Out of	Pts.
1	10	
2	8	
3	8	
4	16	
5	10	
6	10	
7	10	
8	10	
9	6	
Total	88	

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Consider the function f defined by

   $$f(x) = \begin{cases} x^4 - 6, & x < 2 \\ 10, & x = 2 \\ 4x^3, & x > 2 \end{cases}$$

   Which of the following are true statements about this function?

   I. f is continuous at $x = 2$

   II. $\lim\limits_{x \to 2^-} f(x) = f(2)$

   III. f is differentiable at $x = 2$

   IV. $\lim\limits_{x \to 2} f(x)$ exists

   a. II only
   b. I and II only
   c. I and IV only
   d. III only
   e. I, II, III, and IV

   (ii) Oil leaks out of a tank at a rate of $r = f(t)$ gallons per minute, where t is measured in minutes. Which of the following expressions represents the **exact** amount of oil that leaked out of the tank from 15 to 45 minutes after oil started leaking?

   I. $f(45) - f(15)$

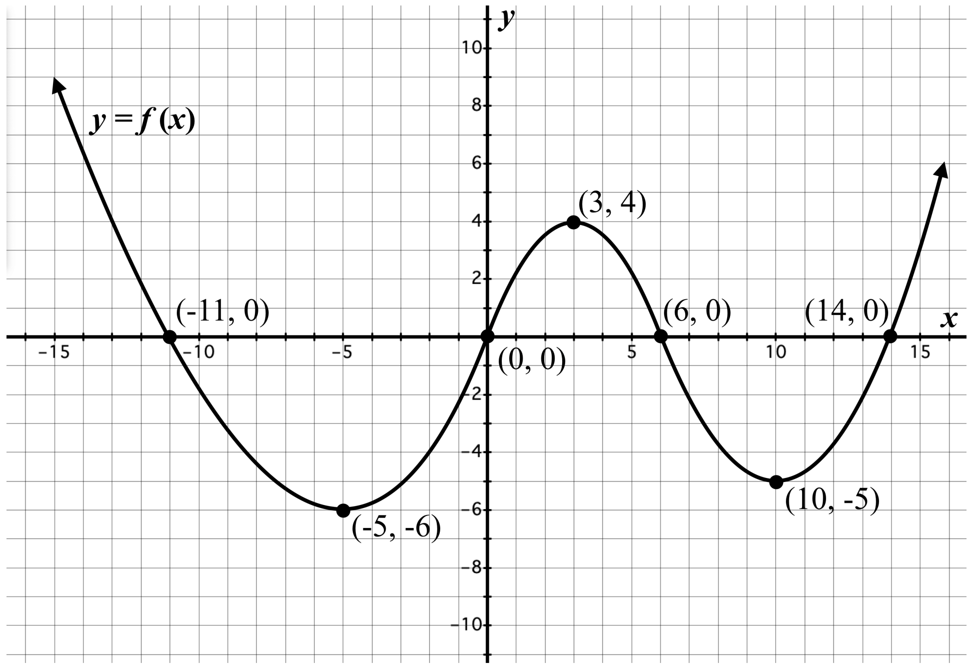
   II. $\displaystyle\int_{15}^{45} f(t)\, dt$

   III. $\displaystyle\sum_{k=1}^{60} f\left(15 + \frac{k}{2}\right) \cdot \frac{1}{2}$

   IV. $\displaystyle\lim_{N \to \infty} \sum_{k=1}^{N} f\left(15 + \frac{30k}{N}\right) \cdot \frac{30}{N}$

   a. I only
   b. I and II only
   c. II and IV only
   d. I, II, and IV only
   e. I, II, III, and IV

(iii) The graph of the function $y = f(x)$ is below. Arrows indicate the end behavior of the function.



On what intervals is $f'(x) > 0$?

a. $(-\infty,\ -5) \bigcup (3,\ 10)$

b. $(-\infty,\ -11) \bigcup (0,\ 6) \bigcup (14,\ \infty)$

c. $(-11,\ 0) \bigcup (6,\ 14)$

d. $(-5,\ 3) \bigcup (10,\ \infty)$

e. $(-\infty,\ 0) \bigcup (6,\ \infty)$

(iv) Let $f(x) > 0$ and $f'(x) > 0$ for $2 \leq x \leq 4$. Which of the following approximations of $\int_2^4 f(x)\,dx$ is the largest?

a. R_4

b. L_4

c. M_4

d. They are all equal.

e. There is not enough information provided to determine which approximation is largest.

(v) Let $f(x) = \sin(2x)$. Which of the following expressions approximates the value of $f(3)$?

a. $f(3) \approx 2\cos(2\pi)$

b. $f(3) \approx 2\cos(2\pi) + \sin(2\pi) \cdot (3 - \pi)$

c. $f(3) \approx \sin(2\pi) \cdot (\pi - 3)$

d. $f(3) \approx 2\cos(2\pi) \cdot (\pi - 3)$

e. $f(3) \approx \sin(2\pi) + 2\cos(2\pi) \cdot (3 - \pi)$

2. (*4 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L'Hôpital's Rule, clearly indicate when you do so and show that the condition for applying L'Hôpital's Rule is satisfied**.

   (a) $\lim_{x\to 1} \frac{x^4-1}{x^3} =$

   (b) $\lim_{\theta\to\pi} \frac{\sin^2(\theta)}{\theta-\pi} =$

3. (*4 points each*) Compute the following derivatives. Do not simplify.

   (a) Let $y = 2^x \cot(\pi x)$. Find $\frac{dy}{dx}$.

   (b) Let $f(x) = \arctan(x^3)$. Find $\frac{d^2 f}{dx^2}$.

4. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

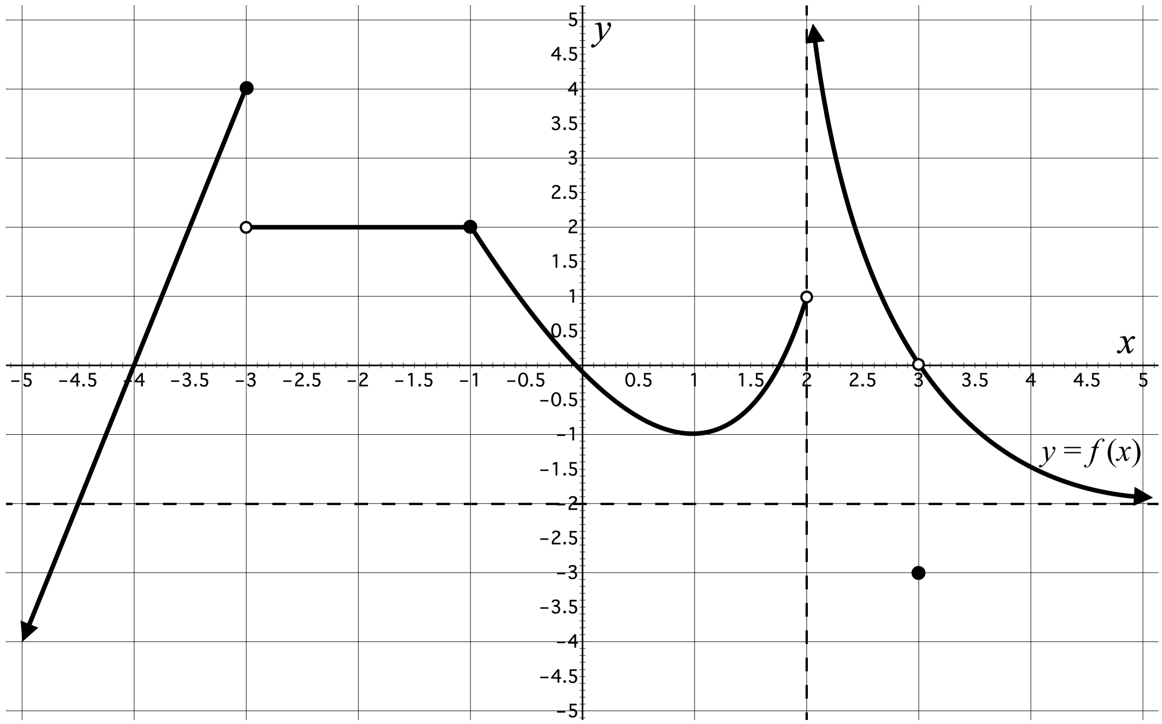
   (a) $\int \left(18x^5 - 10x^4 - \frac{28}{x}\right)\, dx =$

   (b) $\int_1^3 \frac{dt}{t^2} =$

   (c) $\int \theta \sin\left(\theta^2\right)\, d\theta$

   (d) $\frac{d}{dx}\int_1^{x^2} \left(t^5 - 9t^3\right)\, dt =$

5(a) (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all critical points, points of discontinuity, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

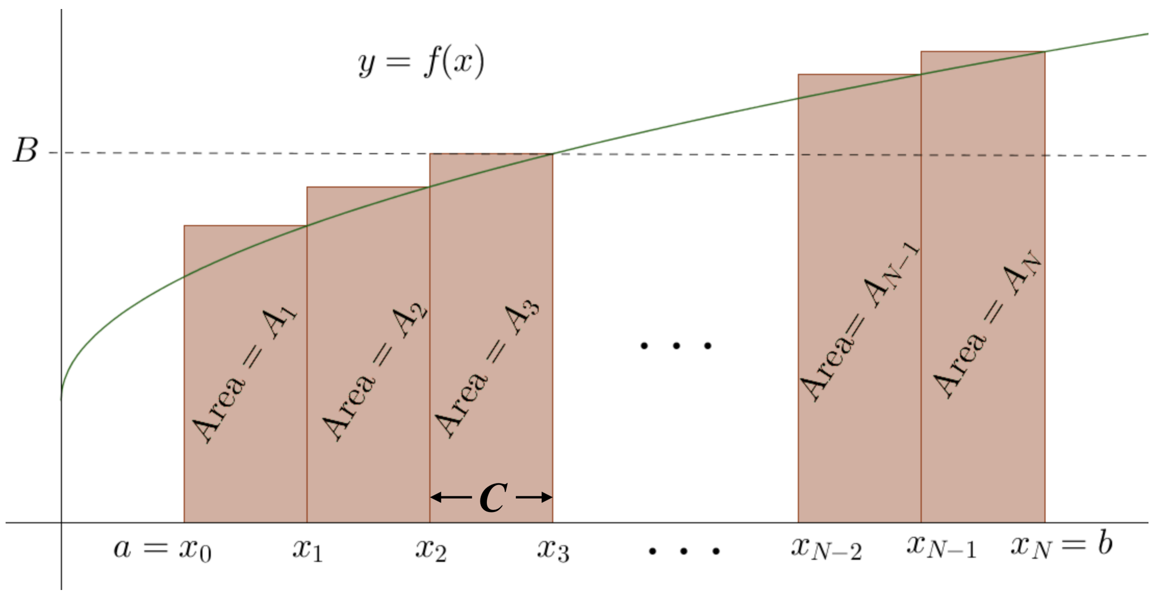
$\lim_{x \to -3} f(x) =$ $\left.\frac{d^2 f}{dx^2}\right|_{x=-4.7} =$ $\int_{-4}^{-1} f(x)\,dx =$

$f'(-3.4) =$ $\lim_{x \to 2^-} f(x) =$ $\lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x} =$

$f'(-1) =$ $\frac{d}{dx} \int_{-2}^{1} f(t)\,dt =$ $\lim_{N \to \infty} \sum_{k=1}^{N} f\left(-2 + \frac{k}{N}\right) \cdot \frac{1}{N} =$

5(b) (*1 point*) Identify a value c in the interval $(-5, 5)$ such that f is discontinuous at $x = c$ and $\lim_{x \to c} f(x)$ exists.

6. (*10 points*) The following image illustrates a Riemann sum using N terms:

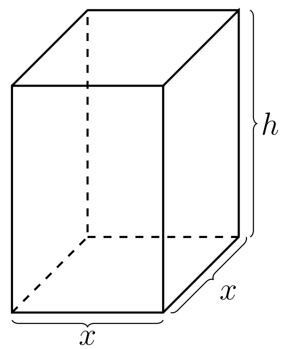


Write each item on the right in the blank next to the corresponding expression on the left. Items B, C, and A_3 refer to the labeled graphical quantities above. ***Each item will be used exactly once.***

Expression	**Matching Item**	**Item**
Δx	____________________	B
$f(x_3)\Delta x$	____________________	C
$f(x_3)$	____________________	A_3
$\sum_{i=1}^{N} f(x_i)\Delta x$	____________________	$\int_a^b f(x)dx$
$\lim_{N\to\infty} \sum_{i=1}^{N} f(x_i)\Delta x$	____________________	$A_1 + A_2 + \cdots + A_{N-1} + A_N$

7. (*10 points*) A child walks at a constant rate of 3 feet per second and lets go of a helium balloon while walking. The balloon rises vertically at a rate of 4 feet per second. At what rate is the distance between the child and the balloon changing 10 seconds after the child let go of the balloon? **You must show your work to receive credit**.

8. (*10 points*) A box of volume 100 in^3 with square bottom and top is constructed out of two different materials. All six sides of the box will be constructed: the cost of the bottom is 4 cents per in^2 and the cost of the top and the sides is 1 cent per in^2. (*Note that the box is not necessarily a cube.*)

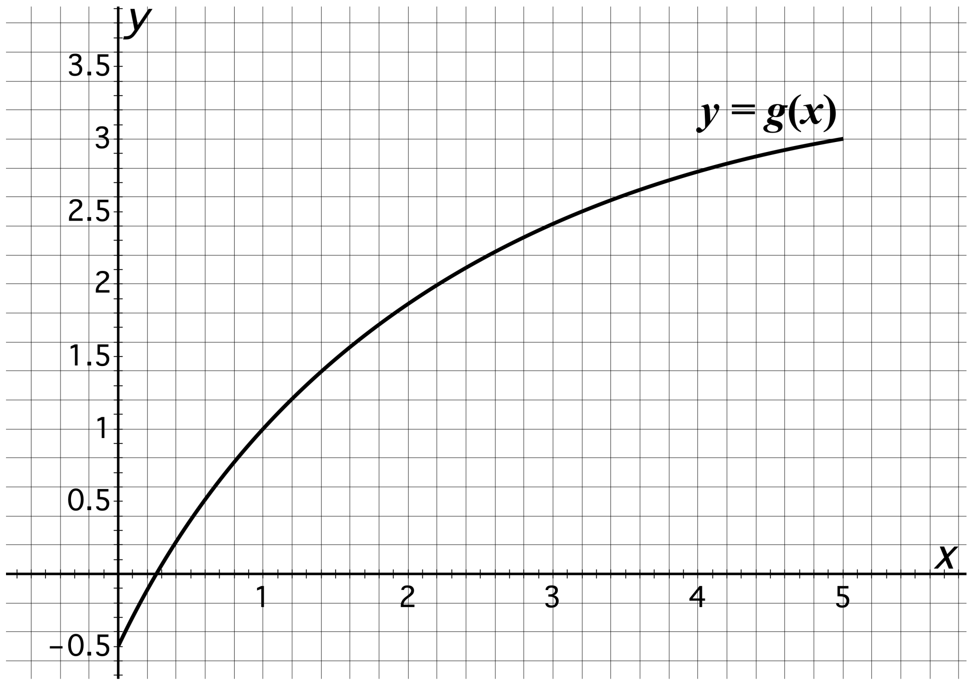


(a) (*3 points*) Write a formula for the **surface area** of the box as a function of a single variable.

(b) (*3 points*) Write a formula for the **cost** of manufacturing the box as a function of a single variable.

(c) (*4 points*) Find the dimensions of the box that minimize total cost. Justify that this is a minimum.

9. (*6 points*) The graph of the function g is given below.



Each of the expressions below represents a numerical value. Identify which of the following expressions represents the largest value and which represents the smallest value. **To receive credit, you must convey your rationale for your selections**.

(i) $g'(1)$

(ii) $\int_1^5 g(x)dx$

(iii) $\sum_{k=1}^{10} g(1+0.4k)\cdot 0.4$

(iv) $g''(1)$

Smallest: ______________________

Largest: ______________________

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BASIC FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$\int \cos(u)\,du = \sin(u) + C$$

$$\int \sec^2(u)\,du = \tan(u) + C$$

$$\int \csc^2(u)\,du = -\cot(u) + C$$

$$\int \sec(u)\tan(u)\,du = \sec(u) + C$$

$$\int \csc(u)\cot(u)\,du = -\csc(u) + C$$

$$\int \tan(u)\,du = \ln|\sec u| + C$$

$$\int \cot(u)\,du = \ln|\sin u| + C$$

$$\int \sec(u)\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc(u)\,du = \ln|\csc u + \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int f\left(u(x)\right)\cdot u'(x)\,dx = \int f(u)\,du$$

$$L_n = \Delta x \sum_{k=1}^{n} f\left(a + (k-1)\Delta x\right)$$

$$R_n = \Delta x \sum_{k=1}^{n} f\left(a + k\Delta x\right)$$

$$M_n = \Delta x \sum_{k=1}^{n} f\left(a + \frac{2k-1}{2}\Delta x\right)$$