Math 2144 Final **Fall 2023** **NAME:** ____________________

CRN: ____________________

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for two 3×5-inch notecards. You may use an approved calculator to perform operations on real numbers, evaluate functions at specific values, and look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e., laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work**, including all steps needed to solve each problem and explain your reasoning to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. This comprehensive exam assesses your understanding of material covered throughout the entire course.

Please write legibly. Your exam will be scanned and graded using an online grading system. The grader will not be able to interpret your work if you do not write clearly and dark enough.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

You have **1 hour** and **50 minutes** to work starting from the signal to begin.

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1. (*2 points each*) Answer the following multiple choice questions. No justification or explanation is required.

   (i) Consider the function f defined by

   $$f(x) = \begin{cases} x^4 - 6, & x < 2 \\ 10, & x = 2 \\ 4x^3 - 16x, & x > 2 \end{cases}$$

   Which of the following are true statements about this function?

   I. f is continuous at $x = 2$

   II. $\lim\limits_{x \to 2^-} f(x) = f(2)$

   III. f is differentiable at $x = 2$

   IV. $\lim\limits_{x \to 2} f(x)$ exists

   a. II only

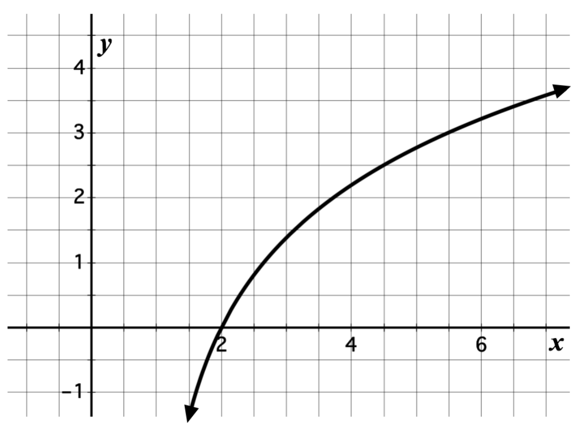
   b. I and II only

   c. I and IV only

   d. III only

   e. I, II, III, and IV

   (ii) The graph of a twice-differentiable function f is shown below. Which of the following is true?



   a. $f(2) < f''(2) < f'(2)$

   b. $f'(2) < f(2) < f''(2)$

   c. $f''(2) < f'(2) < f(2)$

   d. $f''(2) < f(2) < f'(2)$

   e. $f(2) < f'(2) < f''(2)$

(iii) Kacey decides to go for a run before school. She starts her run from home. The function $y = v(t)$ expresses the relationship between Kacey's velocity (in meters per minute) as she runs and the number of minutes elapsed since she started running. What quantity does the following right-endpoint Riemann sum approximate?

$$R_6 = \sum_{k=1}^{6} v\left(1 + 0.5k\right) \cdot 0.5$$

a. The average rate of change of Kacey's velocity over the interval of time from $t = 1$ to $t = \frac{7}{2}$.

b. The change in Kacey's distance away from home over the interval of time from $t = 1$ to $t = 4$.

c. Kacey's acceleration over the interval of time from $t = 1$ to $t = 4$.

d. Kacey's distance away from home after having run for 3.5 minutes.

e. Kacey's instantaneous velocity 3.5 minutes after having left home.

(iv) Suppose $f'(x) > 0$ for all x in the interval $[2, 5]$. The value of the definite integral $\displaystyle\int_2^5 f(x)\,dx$ is **less than** the numerical value of which of the following expressions?

a. $3 \cdot f(4)$

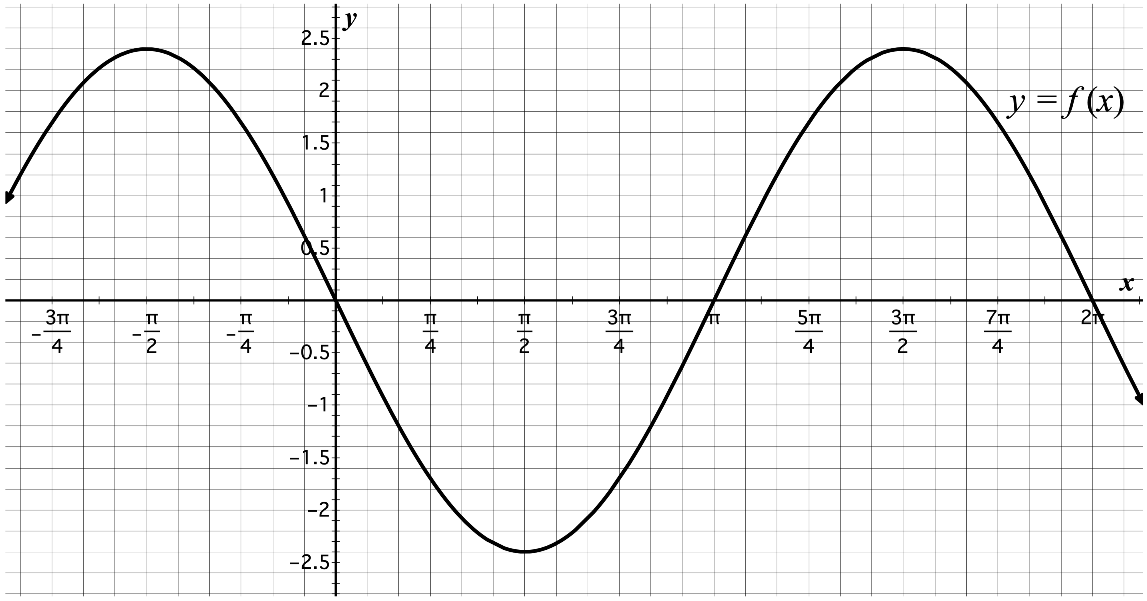
b. $\displaystyle\lim_{N\to\infty} \sum_{k=1}^{N} f\left(2 + \frac{3(k-1)}{N}\right)\frac{3}{N}$

c. $\displaystyle\sum_{k=1}^{N} f\left(2 + \frac{3k}{N}\right)\frac{3}{N}$

d. $\displaystyle\lim_{N\to\infty} \sum_{k=1}^{N} f\left(2 + \frac{3k}{N}\right)\frac{3}{N}$

e. $\displaystyle\sum_{k=1}^{N} f\left(2 + (k-1)\cdot\frac{3}{N}\right)\frac{3}{N}$

(v) The graph of the function f is below.



The expressions below represent numerical values. Which expression represents the largest numerical value?

a. $f'\left(\dfrac{3\pi}{2}\right)$

b. $\dfrac{f\,(2\pi) - f\,(\pi)}{2\pi - \pi}$

c. $f''\left(\dfrac{3\pi}{2}\right)$

d. $\displaystyle\int_{\pi}^{2\pi} f(x)\,dx$

e. There is not enough information provided to compare the numerical values represented by these expressions.

2. (*3 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numerical answers without justification will earn no credit. **If you use L'Hôpital's Rule, clearly justify why you are able to do so**. *Write your final answer in the space provided.*

   (a) $\displaystyle\lim_{x\to 0} \frac{\cos(x)}{x^2+1} =$

   2(a) answer: $\boxed{}$

   (b) $\displaystyle\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}} =$

   2(b) answer: $\boxed{}$

3(a) (*4 points*) Compute the following derivative. **You do not need to simplify**.

Let $g(x) = \dfrac{xe^x}{\tan(x)}$. Find $g'(x)$.

3(b) (*4 points*) Compute the following derivative. **You do not need to simplify**.

Let $f(x) = \pi^{x^2}$. Find the **second derivative** of f, $\dfrac{d^2 f}{dx^2}$.

4(a) (*4 points*) In chemistry, pH is a scale used to measure the acidity of a solution. The pH of a solution is defined by the equation

$$\mathrm{pH} = -\log_{10}(x)$$

where x represents the concentration of hydrogen ions. Compute the instantaneous rate of change of pH with respect to hydrogen ion concentration **when the pH is 2**. **Give an exact answer and include units.** (*Note that* pH *is a single quantity, not the product of* p *and* H.)

Answer: $\boxed{}$

4(b) (*4 points*) The amount of a medication in a person's bloodstream changes at a rate of $h(t) = -700e^{-1.4t}$ milligrams per hour t hours after injecting 500 mg of the medication into the bloodstream. What is the amount of medication in the person's bloodstream 3 hours after taking an initial 500 mg dose? *Round your answer to three decimal places and write your numerical answer in the space provided.*

Answer: $\boxed{}$ mg

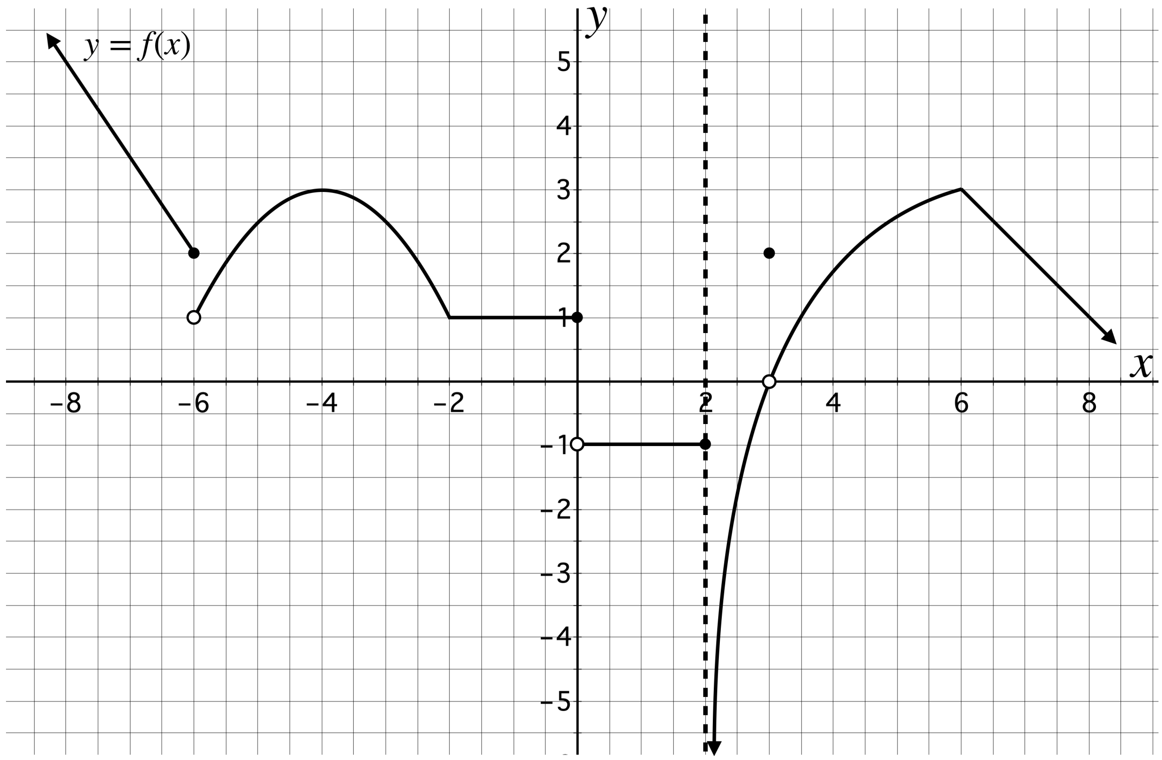
5. (*5 points*) Find $\dfrac{dy}{dx}$ for the curve $x^2 - \sin(y) = xy^2 + \pi^2$. and evaluate $\dfrac{dy}{dx}$ at the point $(\pi, 0)$. *Write your numerical answer in the space provided.*

Answer: $\boxed{}$

6. (*7 points*) Car A is 30 miles north of Car B at 1:00 PM. Car A drives west at a speed of 55 miles per hour and Car B drives west at a speed of 75 miles per hour. How fast is the distance between Car A and Car B changing at 3:00 PM? *Write your numerical answer in the space provided.*

Answer: $\boxed{}$ mi/hr

7(a) (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all critical points, points of discontinuity, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

$\lim_{x\to 6} f(x) =$ $\qquad$ $\left.\frac{df}{dx}\right|_{x=8} =$ $\qquad$ $\int_{-2}^{2} f(x)\,dx =$

$f'(-4) =$ $\qquad$ $\lim_{x\to -6^+} f(x) =$ $\qquad$ $\lim_{\Delta x\to 0} \frac{f(-7+\Delta x)-f(-7)}{\Delta x} =$

$\lim_{x\to 2^+} \frac{df}{dx} =$ $\qquad$ $\lim_{x\to 3} f(x) =$ $\qquad$ $f''(6.3) =$

7(b) (*2 points*) Identify **all values of $\boldsymbol{x}$** in the interval $(-8, 8)$ where f is continuous but not differentiable.

$x =$ $\boxed{}$

8. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, **do not round**.

(a) $\displaystyle\int \left(5x^3 - 7\sec^2(x) + 4e\right)\, dx =$

(b) $\displaystyle\int_1^4 \frac{dt}{9t^2} =$

(c) $\displaystyle\frac{d}{dx}\int_{\pi}^{2^x} \sin^3(\theta)\, d\theta =$

(d) $\displaystyle\int_0^4 \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\, dx =$

9. (*6 points*) Find and classify all local extreme values and inflection points for a function f with the following properties:

$$\frac{df}{dx} = (x-3)^3 (x-5)^2$$

$$\frac{d^2 f}{dx^2} = (x-3)^2 (x-5)(5x-21)$$

Identify local extreme values in the form $f(\blacklozenge)$, where $\blacklozenge$ is the critical point at which the local extreme value occurs. Identify all inflection points in the form $(\bigstar, f(\bigstar))$, where $\bigstar$ is the x-value at which the inflection point occurs.

Local extreme value(s):

Inflection point(s):

10. (*10 points*) What is the radius of a cylinder with minimum surface area and a volume of 200 cubic inches? *Write your numerical answer in the space provided.* ***Round your answer to three decimal places.***

    Note that the surface area (S) and volume (V) of a cylinder are respectively given by $S = 2\left(\pi r^2 + \pi r h\right)$ and $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder.

Answer: $\boxed{}$ inches

11. (*8 points*) The function A is given by

$$A(x) = \int_0^{x^2} \left(\sqrt{t} - 4\right) dt$$

where x is positive. Determine the coordinates of the point of inflection of the graph of $y = A(x)$. *Write the x and y coordinates in the spaces provided.*

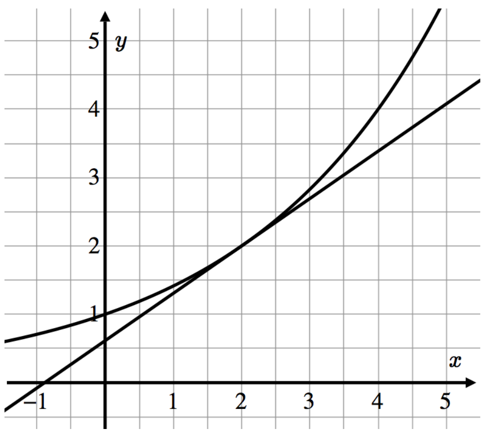
x-coordinate: $\boxed{}$ y-coordinate: $\boxed{}$

12. (*8 points*) The graphs below are of the function g. Write an expression in each blank that represents the numerical value of the quantity identified on the corresponding graph. Select the appropriate expression for each blank from **only** the options below.

$$\frac{d}{dx}\int_2^x g(t)\,dt \qquad \sum_{k=1}^{3} g(1+k)\cdot 1 \qquad \int_1^4 g(x)\,dx \qquad \lim_{h\to 0}\frac{g(2+h)-g(2)}{h}$$

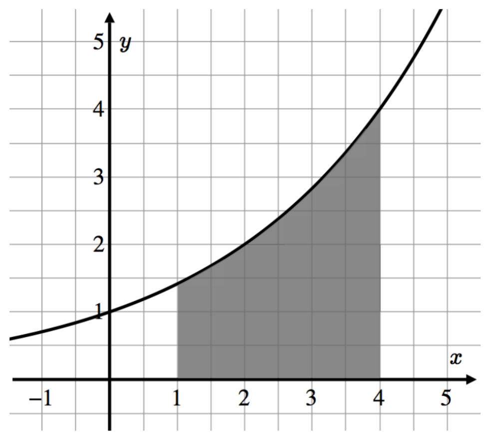
$$\int g(x)\,dx \qquad \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \qquad \sum_{k=1}^{3} g(x)\Delta x \qquad \frac{g(4)-g(0)}{4-0}$$

(a) Slope of the line tangent to the graph of g at $(2, 2)$.



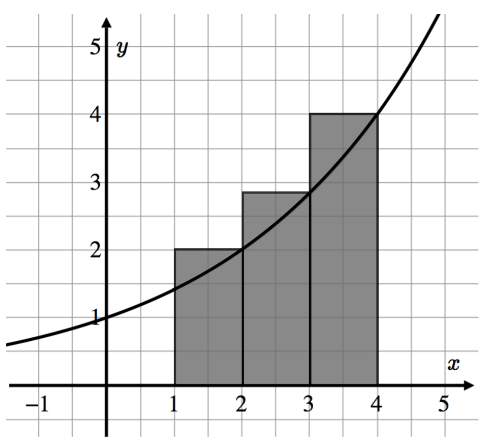
Expression: ____________________

(b) Area bound by the graph of g and the x-axis on the interval $[1, 4]$.



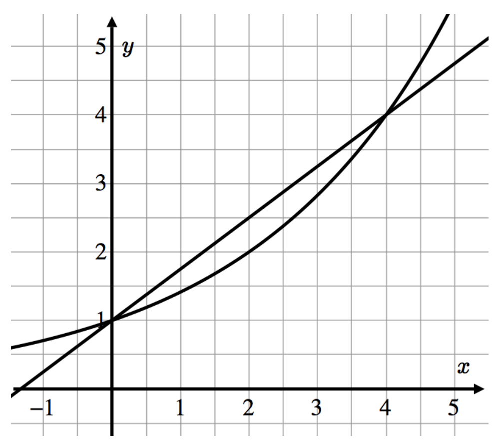
Expression: ____________________

(c) Sum of the area of the rectangles shown below.



Expression: ____________________

(d) Slope of the line secant to the graph of g which passes through the points $(0, 1)$ and $(4, 4)$.



Expression: ____________________

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BASIC FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$\int \cos(u)\,du = \sin(u) + C$$

$$\int \sec^2(u)\,du = \tan(u) + C$$

$$\int \csc^2(u)\,du = -\cot(u) + C$$

$$\int \sec(u)\tan(u)\,du = \sec(u) + C$$

$$\int \csc(u)\cot(u)\,du = -\csc(u) + C$$

$$\int \tan(u)\,du = \ln|\sec u| + C$$

$$\int \cot(u)\,du = \ln|\sin u| + C$$

$$\int \sec(u)\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc(u)\,du = \ln|\csc u + \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int f\left(u(x)\right)\cdot u'(x)\,dx = \int f(u)\,du$$

$$L_n = \Delta x\sum_{k=1}^{n} f\left(a + (k-1)\Delta x\right)$$

$$R_n = \Delta x\sum_{k=1}^{n} f\left(a + k\Delta x\right)$$

$$M_n = \Delta x\sum_{k=1}^{n} f\left(a + \frac{2k-1}{2}\Delta x\right)$$