ABSTRACTS

Equidistribution and Arithmetic Dynamics

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Geometric Discrepancy

by

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Geometric discrepancy quantifies the extent of equidistribution of discrete point sets. Optimal bounds for this quantity strongly depend on the geometry of the underlying family of test sets (axis-parallel rectangles, rotated rectangles, discs, convex sets etc). For example, while for arbitrarily rotated rectangles the minimal discrepancy of an N-point set behaves as a power of N, for axis-parallel rectangles it is only logarithmic (and the exact power of the logarithm is still a subject of debate even on the level of conjectures). We shall explore these phenomena and the influence of geometry on discrepancy estimates, and connections of these problems to probability, number theory, approximation etc. In particular, we shall discuss the behavior of discrepancy with respect to rectangles which are allowed to rotate in a given set of directions ("directional discrepancy").

Number Fields Without Generators of Small Mahler Measure

by

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By the results of Mahler (1964), Silverman (1984) and Ruppert (1998), for any generator α of a number field K of degree $d \geq 2$ over $\mathbb{Q}$ one has $M(\alpha) \gg_d |\Delta_K|^{\frac{1}{2d-2}}$, where Δ_K is the discriminant of the field K, and $M(\alpha)$ is the Mahler measure of α. In the opposite direction, Ruppert asked for which $d \geq 2$ there exists a constant $\kappa(d)$ such that for *every* number field K of degree d over $\mathbb{Q}$ the Mahler measure $M(\alpha)$ of any generator α of K over $\mathbb{Q}$ does not exceed $\kappa(d)|\Delta_K|^{\frac{1}{2d-2}}$. Ruppert himself showed that this is the case for $d = 2$. The case of quadratic extensions was further investigated by Cochrane, Dissnayake, Donohoue, Ishak, Pigno, Pinner and Spencer (2016). On the other hand Vaaler and Widmer (2015) answered Ruppert's question negatively for *composite* $d > 2$. Moreover, using a deep result of Bhargava about quintic fields (2010) they gave a negative answer to Ruppert's question for $d = 5$. Now, we prove that for every *odd* integer $d > 2$ there are infinitely many number fields K of degree d over $\mathbb{Q}$ such that each generator α of K over $\mathbb{Q}$ has Mahler measure at least $d^{-d}|\Delta_K|^{\frac{d+1}{d(2d-2)}}$. Note that the exponent here is strictly greater than $1/(2d-2)$. Thus, this estimate, combined with the above mentioned result of Vaaler and Widmer for composite d, implies that the answer to Ruppert's question is negative for *all* $d \geq 3$. We also show that for each $d \geq 2$ and any $\varepsilon > 0$ there exist infinitely many number fields K of degree d such that every algebraic integer generator α of K has Mahler measure greater than $(1-\varepsilon)|\Delta_K|^{\frac{1}{d}}$. On the other hand, every such field K contains an algebraic integer generator α with Mahler measure smaller that $|\Delta_K|^{\frac{1}{d}}$. This generalizes the corresponding bounds recently established by Eldredge and Petersen for $d = 3$. Our results will appear in *Revista Matemática Iberoamericana*.

Small Height Solutions to Polynomials Systems Linear in Some of the Variables

by

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We consider a system of k multivariate polynomial equations over a number field so that each polynomial is linear in a fixed set of k appropriately separated variables. Provided that a simultaneous solution to this polynomial system exists, we prove the existence of a bounded-height solution. We also describe an explicit algorithm for finding such a solution. In the process, we prove a bound on the height of the inverse of a nonsingular square matrix. We also mention some related results for a single multivariate polynomial linear in at least one of the variables.

Joint work with L. Fukshansky

Misiurewicz Polynomials and Dynamical Units

by

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We study the dynamics of the unicritical polynomial family $f_{d,c}(z) = z^d + c \in \mathbb{C}[z]$. The c-values for which $f_{d,c}$ has a strictly preperiodic post-critical orbit are called *Misiurewicz parameters*, and they are the roots of *Misiurewicz polynomials*. The arithmetic properties of these special parameters have found applications in both arithmetic and complex dynamics. In our recent work, we investigate some new such properties. In particular, we consider the algebraic integers obtained by evaluating a Misiurewicz polynomial at a different Misiurewicz parameter, and we ask when these algebraic integers are algebraic units. This question naturally arises from some results recently proven by several authors, and it is also evocative of the study of dynamical units introduced by Morton and Silverman. We propose a conjectural answer to this question, which we prove in many cases.

Joint work with Rob Benedetto

When is an equidistributed sequence not that well-distributed?

by

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Once we know a sequence is “equidistributed” than what can we deduce? It is tempting to model elements of the sequence as independent random variables and guess how things go from there, in the absence of any evidence to the contrary. For example, we believe that 2 is primitive root for about 37.39% of the primes, and that 3 is also, and that these two events are independent of one another, so is it true that *both* 2 and 3 are primitive roots for 13.98% (the square of 37.39%) of the primes?

In this talk we will discuss this and other aspects of equidistribution that have arisen in recent work of the speaker.

Joint work with (some of) Y. Lamzouri, I. Wigman, K. Soundararajan, Giacomo Cherubini, Alessandro Fazzari, Vítězslav Kala, and Pavlo Yatsynal

Isotriviality, Integral Points, and Primitive Primes in Orbits in Characteristic p

by

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We prove a characteristic p version of a theorem of Silverman on integral points in orbits over number fields and establish a primitive prime divisor theorem for polynomials in this setting - a direct translation of earlier methods is not possible, due to the failure of the Thue-Siegel-Dyson-Roth Theorem. We provide some applications of these results, including a finite index theorem for arboreal representations coming from quadratic polynomials over function fields of odd characteristic. At the heart of our arguments is a detailed comparison of three different notions of isotriviality: for curves, for maps, and for sets of points.

Joint work with Alexander Carney and Thomas J. Tucker

A Study of Shapes

by

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The shape of a lattice is defined to be its equivalence class up to scaling, rotation, and reflection. For a number field, K, there are a few natural lattices that we can ask about the 'shape' of: our primary focus for this talk will be on the shape of a lattice coming from Minkowski's embedding $j : \mathcal{O}_K \to K_{\mathbb{R}}$. Shapes are typically studied as we vary over particular families of fields: most commonly as degree n fields and often with additional Galois restrictions. We highlight a few of the families for which these shapes have been studied and provide an overview of our current projects in this area. Specifically we will discuss our recent work on the shape of pure, prime degree, number fields including the (regularized) equidistribution of these shapes. We will also mention joint work with Rob Harron in which we define a natural refinement of shapes to study the case of non-Galois sextic fields (i.e. those with absolute Galois group $C_3 \wr C_2$): here we prove equidistribution results and observe a relationship between the study of shapes and the log terms in Malle's conjecture. Time permitting we will discuss ongoing work regarding the shape of another natural lattice associated to number fields, the unit-log lattice.

Explicit Algebraic Equations for Dynatomic Cycles

by

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The n-th dynatomic polynomial construction for morphisms of $\mathbb{P}^1$ is a well known generalization of the n-th cyclotomic polynomial and produces a single polynomial whose roots are (generically) the points with exact period n. Using instead the intersection of the graph variety and the diagonal in $\mathbb{P}^N \times \mathbb{P}^N$, this construction can be generalized to endomorphisms of higher dimensional projective spaces and produces an effective zero-cycle. Again, (generically) the points in support of this n-th dynatomic cycle are the points of exact period n. We present methods to compute algebraic equations that describe the dynatomic cycle as a variety.

Joint work with Asma Zangana (Saint Louis University)

Equidistribution in Families of Dynamical Systems

by

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I will describe the natural extension of my equidistribution result on families of abelian varieties (arXiv:2101.10272) to the setting of families of dynamical systems, giving also some background on Arakelov geometry. This (unpublished) extension, which was also independently obtained in work of Yuan–Zhang (arXiv:2105.13587) and Gauthier (arXiv:2105.02479), was in fact also a central motivation for my original work.

Stability of Certain Higher Degree Polynomials

by

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We study the stability of $f(z) = z^d + \frac{1}{c}$ for $d \geq 3$, $c \in \mathbb{Z} \setminus \{0\}$. We show that whenever $f(z)$ is irreducible, all its iterates are irreducible over $\mathbb{Q}$, that is, $f(z)$ is stable over $\mathbb{Q}$ for infinite values of d.

Joint work with R. Sarma and H. Sharma

Several Aspects of Dynamical Mahler Measure

by

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The Mahler measure of a multivariable polynomial or rational function P is given by the integral of $\log|P|$ where each of the variables moves on the unit circle and with respect to the Haar measure. We consider a dynamical generalization of Mahler measure and present dynamical analogues of various results from the classical Mahler measure as well as examples of formulas allowing the computation of the dynamical Mahler measure in certain cases.

Joint work with Carter, Manes, Miller, and Mocz

The Dynamical Bogomolov Conjecture in Families of Split Maps

by

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Inspired by an analogy between torsion and preperiodic points, Zhang has proposed a dynamical generalization of the classical Manin-Mumford and Bogomolov conjectures. A special case of these conjectures, for 'split' maps, has recently been established by Nguyen, Ghioca and Ye. In particular, they show that two rational maps have at most finitely many common preperiodic points, unless they are 'related'. In this talk we discuss uniform versions of the dynamical Bogomolov conjecture across 1-parameter families of split maps and curves. To this end, we establish instances of a 'relative dynamical Bogomolov conjecture'. Tools in our proof include an arithmetic equidistribution theorem, established recently by Yuan and Zhang, as well as complex analytic results.

Joint work with Harry Schmidt (University of Basel)

Intersecting Varieties and Multiplicative Subgroups over Fields with the Bogomolov Property

by

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In this talk, we aim to recall some bounded height results on unlikely intersections of algebraic varieties in the torus, and discuss how some statements of finiteness can be concluded when the base field has the so called Bogomolov property.

Basins of Infinity and Böttcher Maps

by

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Can we determine the complex polynomial dynamics from the dynamics in the basin of infinity in $\mathbb{C}$? An (obvious) answer is no. Now we restrict the coefficients to be algebraic and consider all the archimedean and nonarchimeadean basins of infinity, we may ask if the polynomial dynamics is determined by the dynamics in all the above basins of infinity. I will talk about the case of unicritical polynomials.

Joint work with H. Fu

On Totally Real PCF Parameters

by

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An algebraic number is called totally real if its minimal polynomial splits completely over $\mathbb{R}$. In this talk, I will discuss a finiteness result of totally real algebraic numbers c such that the uncritical polynomial $f_c(x) = x^2 + c$ is **P**ost-**C**ritically **F**inite (i.e., the orbit of 0 under f_c-iteration is finite). This is based on joint work (in preparation) with Clayton Petsche. Our approach relies upon potential-theoretical features of the classical Mandelbrot set and a property of zeros of a monic Jacobi polynomial.

Joint work with Clayton Petsche

An Inner Product on Adelic Measures

by

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We define an inner product on a vector space of adelic measures over a number field. When restricted to the subspace of adelic measures with total mass zero, the norm induced by our inner product agrees with the mutual energy pairing considered by Favre & Rivera-Letelier and by Fili. We find that the norm of the canonical adelic measure associated to a rational map is commensurate with the Arakelov height on the space of rational functions with fixed degree. As a consequence, the Arakelov–Zhang pairing of two rational maps f and g can be bounded from below as a function of g.

Joint work with Clay Petsche

Diophantine Approximation on Conics

by

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Given a conic $\mathcal{C}$ over $\mathbb{Q}$, it is natural to ask what real points on $\mathcal{C}$ are most difficult to approximate by rational points of low height. For the analogous problem on the real line (for which the least approximable number is the golden ratio, by Hurwitz's theorem), the approximabilities comprise the classically studied Lagrange and Markoff spectra, but work by Cha–Kim and Cha–Chapman–Gelb–Weiss shows that the spectra of conics can vary. We provide notions of approximability, Lagrange spectrum, and Markoff spectrum valid for a general $\mathcal{C}$ and prove that their behavior is exhausted by the special family of conics $\mathcal{C}_n : XZ = nY^2$, which has symmetry by the modular group $\Gamma_0(n)$ and whose Markoff spectrum was studied in a different guise by A. Schmidt and Vulakh. The proof proceeds by using the Gross-Lucianovic bijection to relate a conic to a quaternionic subring of $\mathrm{Mat}^{2\times 2}(\mathbb{Z})$ and classifying invariant lattices in its 2-dimensional representation.

Equidistribution Towards the Activity Measures in Non-Archimedean Parameter Curves

by

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In this talk, let f be an analytic family of endomorphisms f_t of degree $d > 1$ of the Berkovich projective line $\mathbb{P}^{1,\mathrm{an}} = \mathbb{P}^{1,\mathrm{an}}_K$ defined over an algebraically closed field K that is complete with respect to a non-trivial and non-archimedean absolute value, parametrized by t in a domain (i.e., a connected open subset) V in the Berkovich analytification C^{an} of a smooth projective algebraic curve C defined over K.

To the pair (f, a) of the above analytic family f and each analytically marked point $a = a(t)$ in $\mathbb{P}^{1,\mathrm{an}}$ parametrized by t in V (i.e., a is a morphism from V to $\mathbb{P}^{1,\mathrm{an}}$), setting

$$a_n(t) := f_t^n(a(t)),\ t \in V, \quad \text{for each } n \in \mathbb{N}$$

so that a_n is a morphism from V to $\mathbb{P}^{1,\mathrm{an}}$, the (possibly trivial) activity measure

$$\mu_{(f,a)} := \lim_{n\to\infty} \frac{(a_n)^*\delta_{\xi_g}}{d^n} \quad \text{weakly on } V$$

is associated, where ξ_g is the Gauss (or canonical) point in $\mathbb{P}^{1,\mathrm{an}}$, and δ_ξ is the Dirac measure on $\mathbb{P}^{1,\mathrm{an}}$ at each point $\xi \in \mathbb{P}^{1,\mathrm{an}}$.

Then the following equidistribution result in the parameter space V holds; for every $\xi \in \mathbb{P}^{1,\mathrm{an}}$ but a subset of (logarithmic) capacity 0, we have

$$\lim_{n\to\infty} \frac{(a_n)^*\delta_\xi}{d^n} = \mu_{(f,a)} \quad \text{weakly on } V.$$

The proof is based on a Nevanlinna theoretic argument, and is standard when V is separable in that V contains a countable dense subset. In general, we use a skeleton Σ of the augmented Berkovich curve C^{an} of V, which is a finite subgraph in C^{an} so that all connected components of the complement of Σ in C^{an} are open (Berkovich) balls in C^{an}.

Joint work with Reimi Irokawa (Tokyo Institute of Technology)

Uniform Boundedness for Preperiodic Points of $x^d + c$

by

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Arithmetic dynamics is a combination of dynamical systems and number theory. In this talk, we discuss the rational preperiodic points of polynomial $x^d + c$. Under the assumption of the *abc*-conjecture, we prove that $x^d + c$ has no rational periodic point of exact period greater than 1 for sufficiently large integer d and $c \neq -1$.

Sato-Tate Conjecture in Arithmetic Progressions for Certain Families of Elliptic Curves

by

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In this talk we will study moments of the trace of Frobenius of elliptic curves if the trace is restricted to a fixed arithmetic progression. In conclusion, we will obtain the Sato-Tate distribution for the trace of certain families of Elliptic curves. As a special case we will recover a result of Birch proving Sato-Tate distribution for certain family of elliptic curves. Moreover, we will see that these results follow from asymptotic formulas relating sums and moments of Hurwitz class numbers where the sums are restricted to certain arithmetic progressions.

Joint work with Kathrin Bringmann and Ben Kane

On the p-adic Distribution of CM Points on the Modular Curve

by

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Singular moduli are the j-invariants of CM points in the moduli space of elliptic curves. These numbers lie at the heart of the theory of abelian extensions of imaginary quadratic fields. We will discuss the p-adic asymptotic distribution of CM points, and, as an application, the finiteness of singular moduli that are S-units. This complements recent works by Bilu, Habegger and Kühne and by Li.

Joint work with Sebastian Herrero and Ricardo Menares

The Crucial Set in Non-Archimedean Dynamics: its History and Applications

by

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Let K be a complete, algebraically closed, non-archimedean valued field, and let f be a rational function in $K(z)$ of degree $d > 1$. The crucial set of f is a finite, nonempty set of points belonging to the non-classical part of the Berkovich projective line over K. It consists of at most $d - 1$ points, and it carries a natural probability measure, the "crucial measure". The barycenter of the crucial measure is the minimal resultant locus of f, which determines the point corresponding to f in the moduli space of rational functions of degree d. The crucial measures of the iterates of f converge weakly to the canonical measure supported on the Berkovich Julia set of f.

This talk will cover the origins and geometric meaning of the crucial set, and its generalization by Okuyama. It will discuss the equidistribution theorems of Jacobs, Okuyama, and Okuyama-Nie. It will conclude with an application to the moduli point of a quadratic rational function by Doyle, Jacobs and Rumely, and the striking recent application by Okuyama and Nie showing the asymptotic GIT-stability of the moduli of the iterates of a non-archimedean polynomial.

Rigidity and Unlikely Intersections for Stable p-adic Dynamical System

by

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Let K be a finite extension of the p-adic field $\mathbb{Q}_p$, with ring of integers $\mathcal{O}_K$ and unique maximal ideal $\mathfrak{m}_K$. A stable p-adic dynamical system $\mathcal{D}$ over $\mathcal{O}_K$ is a collection of p-adic power series in $\mathcal{O}_K[[x]]$ without constant term such that the power series commutes with each other under formal composition. For example, if F is a formal group of finite height over $\mathcal{O}_K$, then the endomorphism ring $\mathrm{End}_{\mathcal{O}_K}(F)$ of F is a stable p-adic dynamical system. Laurent Berger studied to what extent the torsion points $\mathrm{Tors}(F)$ of a formal group F over $\mathcal{O}_K$ determines the formal group. He proved that if $\mathrm{Tors}(F_1) \cap \mathrm{Tors}(F_2)$ is infinite then $F_1 = F_2$. He further asked the question, if $\mathcal{D}$ is a stable p-adic dynamical system, then: to what extent the set $\mathrm{Preper}(\mathcal{D})$ of preperiodic points of power series in $\mathcal{D}$ determines $\mathcal{D}$? In our work we have answered this question with the help of a Conjecture due to Laurent Berger, that was originally conjectured by Jonathan Lubin. We have proved if $\mathcal{D}_1$ and $\mathcal{D}_2$ are two stable p-adic dynamical systems over $\mathcal{O}_K$ such that $\mathrm{Preper}(\mathcal{D}_1) \cap \mathrm{Preper}(\mathcal{D}_2)$=infinite, then $\mathcal{D}_1 = \mathcal{D}_2$.

Joint work with Absos Ali Shaikh

Divisibility Conditions on the Order of the Reductions of Algebraic Numbers

by

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In 1967 Hooley proved (under GRH) Artin's conjecture on primitive roots: for any $g \in \mathbb{Z} \setminus \{-1, 0, 1\}$ which is not a square, there are infinitely many primes p such that g is a primitive root modulo p (i.e. the multiplicative order of ($g \bmod p$) equals $p-1$). We consider some variations of this problem over number fields for the order of the reductions of algebraic numbers. Let K be a number field and let G be a finitely generated group of algebraic numbers. We investigate the primes $\mathfrak{p}$ of K (up to discarding finitely many of them) such that the order of ($G \bmod \mathfrak{p}$) is divisible by a given integer, or more generally such that it lies in a given arithmetic progression. Our results generalize previous work by Pappalardi and by Ziegler.

Points on Tri-Involutive K3 Surfaces

by

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Let W be a surface in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$ given by the vanishing of a $(2,2,2)$ form. The three projections $W \to \mathbb{P}^1 \times \mathbb{P}^1$ are double covers that induce three non-commuting involutions on W. Let G be the group of automorphisms of W generated by these involutions. We investigate the G-orbit structure of the points of W. In particular, we study G-orbital components over finite fields and finite G-orbits in characteristic 0.

Joint work with Elena Fuchs, Matthew Litman, and Austin Tran

Fourier Analysis, Discrepancy, and Koksma Inequalities on the p-adic Integers

by

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In this talk, we consider equidistribution on the p-adic integers $\mathbb{Z}_p$. In particular, we can set up Fourier Analysis on $\mathbb{Z}_p$ using its dual group the Prufer p-group. By using Parseval's Theorem, we can obtain an analogue of the LeVeque inequality on the discrepancy of sequences in $\mathbb{Z}_p$. We show that the Fibonacci sequence is equidistributed in $\mathbb{Z}_p$, and derive a bound on its discrepancy. We also show how one can derive a Fourier Analytic Koksma inequality on $\mathbb{Z}_p$.

Equidistribution in Stochastic Dynamical Systems

by

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Given a set of rational maps on the projective line, endowed with a probability measure, we study the dynamics of this stochastic family. As these families may not be defined over a single number field, we introduce the concept of a generalized adelic measure which extends notions introduced by Favre and Rivera-Letelier and Mavraki and Ye. We prove that, under some assumptions on boundedness, the heights for stochastic families of rational maps are Weil heights, associated to generalized adelic measures. We prove an equidistribution theorem for generalized adelic measures, and apply this to prove an equidistribution result for random backwards orbits in stochastic arithmetic dynamics.

Joint work with John R. Doyle and Paul Fili

A Tits Alternative for Rational Functions

by

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The Tits alternative states that any finitely generated linear group either contains a free group on two generators or has a solvable group of finite index. We prove some variants of the Tits alternative for semigroups of rational functions. In particular, we show that in characteristic 0 there are no semigroups of rational functions of intermediate growth (that is, all have either polynomial or exponential growth).

A Schauder Basis for the Multiplicative Group of a Number Field

by

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Let k be an algebraic number field, $k^\times$ the multiplicative group of nonzero elements in k, and let $\mathcal{G}_k$ denote the multiplicative group $k^\times$ modulo its torsion subgroup. Then $\mathcal{G}_k$ is a free abelian group. We construct a countable set of algebraic numbers in $k^\times$ which are coset representatives of a basis for $\mathcal{G}_k$. When $\mathcal{G}_k$ is injected into a real Banach space $\mathcal{X}$ with norm equal to the Weil height of algebraic numbers, the image of the basis forms an unconditional Schauder basis for the closed linear subspace of $\mathcal{X}$ spanned by $\mathcal{G}_k$. This leads to a necessary and sufficient condition for a homomorphism $\varphi : \mathcal{G}_k \to \mathbb{Z}$ to extend to a continuous, linear functional on $\mathcal{X}$.

Torsion Values of Sections on Elliptical Billiards and Diophantine Problems in Dynamics

by

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We shall consider "sections" of elliptic families, and especially their "torsion values". For instance, what can be said of the complex numbers b for which $(2, \sqrt{2(2-b)})$ is torsion on the Legendre curve $y^2 = x(x-1)(x-b)$? In particular, we shall recall results of "Manin-Mumford type" and focus to illustrate some applications to elliptical billiards. Finally, if time allows we shall frame these issues as special cases of a general question in arithmetic dynamics, which can be treated with different methods, depending on the context.

(Most results refer to work with Pietro Corvaja and David Masser.)